A Companion to

Mathematical Modeling

Applications with GeoGebra

Task Annotations

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Table of Contents

Chapter 2 Page 20

Chapter 3 Page 27

3.8 The Rule of 72

Rating: 4
Keywords: Algebra, rational function, exponential function, log function, comparison, money, interest rate, doubling time

3.9 Fish Growth Rates

Rating: 3.7
Keywords: rates, exponential growth, variables

Chapter 4 Page 36

4.1 Cooling Activity 2

Rating: 3
Keywords: compare and contrast, error, law of cooling, functions, exponential function, polynomial, fourth degree polynomial, sixth degree polynomial

4.2 Body Surface Area

Rating: 4
Keywords: functions, units, body mass index, surface area, empirical formulas, data analysis

4.3 Warm Blooded Animals: Finding heat production

Rating: 3
Keywords: graphing, logarithmic function, heat output, best fit

4.4 Control of Insect Pests

Rating: 2.5
Keywords: functions, multiple representations, growth, power function, fit

4.5 Selling Magazines for Christmas

Rating: 5
Keywords: model, sales, piecewise functions, composition functions, algebra

4.6 Determining the center and size of a tumor

Rating: 4.3
Keywords: geometry, circles, area, center, error, sum of squared error

4.7 FREE FALL

Rating: 4
Keywords: exponential function, data analysis, best fit, freefall, functions, validity

4.8 Concentration

Rating: 3
Keywords: concentration, statistics, maximum, formula, function, models, best fit, compare and contrast, multiple representations

4.9 Air current measured over tip of an airplane, using fluid dynamics

Rating: 2.3
Keywords: current, fluid dynamics, parameters, best fit, line of best fit, error, sum of squared errors, base equation

4.10 Tides

Rating: 4
Keywords: tides, water level, function, error, sine, trigonometry, best fit, residuals, sum of squared errors

Chapter 5 Page 50

5.1 The Fish Farm II

Rating: 3
Keywords: linear function, quadratic function, cubic function, maximum, revenue, cost, profit

5.4 The Airplane Wing

Rating: 3
Keywords: constants, cross-sectional, symmetric, area, integration, reflection, polynomial fit

5.6 Volume of a Pear

Rating: 3
Keywords: volume, 3D, roots, functions, integration, displacement, scale

5.8 Exercise: Determining oxygen uptake from power

Rating: 4
Keywords: curve, area, integration, functions, free-form

5.9 Bicycle Reflectors

Rating: 4
Keywords: movement, circles, distance, circumference, distance

5.10 Cardiac Output

Rating: 4
Keywords: concentrations, polynomial, fit, area, integration, change, linear function, logarithmic function, polynomial function

5.11 Medication

Rating: 3.5
Keywords: area under the curve, function of time

5.13 Temperature Change

Rating: 3
Keywords: temperature, periodic change, trigonometric functions, Fourier series

Chapter 6 Page 64

6.1 Time Taken for Water to Chill Out

Rating: 3
Keywords: temperature, cooling, flow, predictions, extension, variables, Newton's Law of Cooling

6.2 Moose Hunting

Rating: 3
Keywords: population, carrying capacity, Verhulst Model, ordinary differential equation (ODE), fit

6.4 Skydiving

Rating: 4
Keywords: maximum, velocity, freefall, differential equations, integral, area, sliders

6.5 Flu Epidemics

Rating: 4
Keywords: SIR model, differential equations, variance

6.6 USA's Population

Rating: 3
Keywords: population, quadratic, exponential, logistic

6.7 Cheetahs vs Antelope: A Game of Survival

Rating: 3
Keywords: differential equations, difference equations, cyclic model, numerical solutions, making predictions, validity

6.8 Amount of smoke left in a smoking break room

Rating: 4
Keywords: differential equations, parameters

6.9 Alcohol consumption and sobriety over time

Rating: 4.3
Keywords: differential equations, rate of change, parameters

Chapter 7 Page 79

7.1 The Looping Pen

Rating: 3
Keywords: motion, related rates, circles, segments

7.2 Comparing Areas

Rating: 3
Keywords: polygons, areas, triangles, intersection, proof, central area

7.3 Explore relationships between the heights, medians, circumscribed, and inscribed circles of the four triangles constructed with four intersecting lines

Rating: 4
Keywords: exploration, GeoGebra tools, triangle, medians, heights, circumscribed circle, and inscribed circles

7.4 The points on a triangle and how they are related to each other (an interactive figure) including the Euler line

Rating: 4.3
Keywords: centers, triangle, Euler line, incenter, centroid, circumcenter, orthocenter, nine-point center, symmedian point, gergonne point, nagel point, fermat point

7.5 Trisected Area

Rating: 3
Keywords: triangles, geometry, area

7.6 Spirograph

Rating: 4
Keywords: spirograph, radius, gears, multiple representations, proof

7.7 Studying geometric oscillatory drawings with the Harmonograph

Rating: NR
Keywords: harmonograph, circles, lines, intersections, locus, oscillatory motion

7.8 Folding Paper

Rating: 3
Keywords: right triangle, triangles, maximum, folding, area

7.9 The Locomotive

Rating: 3.5
Keywords: circles, trigonometry, sine, cosine, rotation, segments, periodic motion

Chapter 8 Page 94

8.3 Determining the endangeredness of the Red Squirrel

Rating: 3.7
Keywords: Markov chains, transitional matrices, recursive relationship, probability

8.4 Chlorine

Rating: 3
Keywords: geometric series, functions, fit, convergence

8.5 The Deer Farm

Rating: 3
Keywords: model, population, recursive rule, stable population

8.6 Analyzing a Number Sequence

Rating: 4
Keywords: sequences, series

8.7 Inner Areas in a Square

Rating: 1
Keywords: square, inner square, ratio, area, inner polygons

8.8 Ratio of Inner and Outer Areas in a Triangle

Rating: 3.7
Keywords: proportion, ratio, discrete, discreteness, triangles, construction, area, hexagons, geometry

8.9 A Climate Model Based on Albedo

Rating: 4
Keywords: discrete, maximum, minimum, temperature

8.10 Traffic Jam

Rating: 4
Keywords: statistics, trigonometry, waves, circle, density

8.11 Wildfire

Rating: 3
Keywords: code, command buttons, time, direction, probability, variables, meta-scripts

Chapter 1

Some Introductory Problems

Sam Barra

Task 1.1: Ticket price and revenue based on number of passengers on a plane

Keywords

linear functions, break-even point, intersection

Content Level: Algebra 1

Overall: 3.5
Engaging: 3- The topic could be fairly interesting since many students associate flying on planes with going on a vacation or fun trip- would be good to do around spring break time.
Difficulty: 2- Fairly simple problem even with GeoGebra.
Math Content: 4- Very commonly seen problem that has application to many different problems of similar types.

Objectives:
- Revenue/profit functions, finding the intersection of two lines
- Beginning use of GeoGebra- making tables, using graphs to find intersections

Discussion: This kind of problem made me think about many of the problems that were done in Algebra where you're trying to find the break-even point. Different types of contexts for this problem could include tickets being sold at a movie theater, prices at a store in order to make a profit, etc. This kind of problem is a good starting problem for people who are new to GeoGebra since it is done in a very similar manner to Excel which students have probably already used. The second proposed solution only involves making a graph which is less steps as well.

Anticipated Student Difficulties: Not too many issues as all. The equation is fairly easily obtained by using substitution. The most difficult part would probably be using the technology in order to graph the function or just making sure students know that they are trying to find the intersection.

Extensions: Very common type of math problem seen in different algebra classes
Alternate Uses Mathematically: Could lead into talks about profit functions as well

What you learned: At first glance of the problem, I thought that it looked exactly like something I would do in Excel but one cool thing I learned that was directly from the text is "The GeoGebra spreadsheet can handle all kinds of objects in its cells, while most other spreadsheets only handle real numbers, text, and formulas".

Megan Campbell
Gadea Ortiz

Task 1.2: Using various models to find how long the pasture lands in a field will provide for a certain number of cows.

Keywords

Graphing, functions, linear function, exponential function

Content Level: Algebra 1, connecting multiple functions together

Overall: 3-stars
Engaging: 2-stars **Difficulty:** 4-stars Math **Content:** 3-stars

Objectives:

- Graphing
- Types of functions (linear, exponential, etc.)
- Practicality: How valid their model is (strength & weaknesses)
- Task explicitly asks students to model the question given
- Technology is optional.
- Can be modeled algebraically and applied to a graph using technology (calculators, software, etc.)

Discussion: This task gives two pieces of information for students and asks them to model and make predictions based on what they had. As a result this forces students to try to think of multiple ways to represent the information (linear or exponential or otherwise). Students who model it in different ways will get different results and this will force them to question the differences in their results and which is accurate.

Anticipated Student Difficulties: Some students might struggle with deciding what model to use to represent the situation and how the information affects the model. Additionally some students may struggle to see the validity of another student's method after they create a method themselves.

Extensions: Give them more information so there is an answer that is more ideal. This problem itself has no "correct" answer based on how it is set up. If there is more information or an additional question that could lead the result a certain way that may be helpful. This task can lead to the discover of nonlinear relationships. After the linear equations the teacher can give this to the students to model and then they can talk about the strength and weakness of the model and introduce nonlinear relationships to the students.

Ideas about Implementation: I would recommend changing the topic to relate to students better. The topic of cows in a pasture isn't something that many students can relate to especially in an urban setting. If this related to how many students could the school cafeteria feed or something that might be more engaging.

Cayla Pearsall

Task 1.3: A Bit of Chemistry, using concentration percentages

Keywords

Chemistry, concentration, percentage, equations

Class Level: Algebra 1

Overall: 3.33/5
Engaging: 4/5
I feel that this task would be very engaging to a student interested in Chemistry, however this could be remedied by modifying the task.
Difficulty: 2/5
I feel that when using GeoGebra this task would not be very difficult for a student that is in an algebra classroom.
Math Content: 4/5
I feel that this is all math content, however it is geared toward a science class.

Objectives:
- Creating algebraic equations when given percentages and solution amounts.
- Student should be able to model creating a new concentration of a solution using algebraic equations.
- Student could use Geogebra after attempting to solve the problem without technology.

Discussion: This task could relate to cooking recipes, and converting them to double batches or triple batches, or even downsizing a recipe to fit what you have on hand. I immediately thought of using this as a word problem for middle school students, this problem would be given as a challenge question when a group or individual finishes early.

Anticipated Student Difficulties: Students may not understand that when combining the two different concentrations you get a whole new concentration. Instead they may think you can add water to "dilute" the stronger concentration.

Items that Stand Out: I feel that if you let the students attempt to use Geogebra to solve this problem many of them will not be able to figure out how to use it. Even with the explanation given for the second proposed solution it was still very complicated to do in Geogebra. Students may not understand the Math involved in the problem if they go right to using technology.

Extensions: Chemistry classrooms
Alternate Uses Mathematically: You could give the students the equation and have them solve it for practice in a beginning algebra classroom.
Ideas about Implementation: Have the students start by solving it without technology. Then have them use Geogebra to see if they can create a way of "solving" the problem, for various solutions.
Experiment: Could use dyes in water in different concentrations.

What you learned: http://www.chemcollective.org/chem/ubc/exp03/tutor3.php

Karlie Goretski
Sakthi Shanmugasundaram

Task 1.5: Perspective: “Your perception will be different depending on how you look at it.”

Keywords
measuring, distance, perception, image size

Overall: 3
Engaging: 3 (It has the students involved when measuring distances and working with the math tools)
Difficulty: 2 (As long as the student is following the steps it is not that difficult)
Math Content: 3 (Works with the distance equation)

Objectives:

- Students can look at the same object at different distances and make connections between the distances and the object.
- Students can measure and be involved in the task thus to be able to engage with perception hands on.
- GeoGebra is the technology required.

Discussion: This task walks through students on how to find the distances from each image and how each distance will be different. It also focuses on how the image size should not change due to the fact the distances are different from a certain point. I think this modeling task works along with the example with the side window on a car. This task reminded me of how the car window has the description “things appear closer than they seem.” Thus this task would be a good idea to help students with perception.

Anticipated Student Difficulties: It may be take the students awhile to figure out how to experiment with this idea on their own. I think that an example will help students show them which direction to go to. Also, perception as a concept may be hard to visualize so letting them do a real life modeling task can grab their understanding. Also, using GeoGebra with this task may be harder for students who are using GeoGebra for this first time.

Ryan Gronwick

Task 1.6: Finding Lake Erie's Area Using Technology Such As a Print-Out Of Picture Of The Lake and the GeoGebra Application.

Keywords:
Area, scale, units, density, volume

Overall: 4 stars
Engaging: 5 stars **Difficulty:** 4 stars **Math Content:** 3 stars

Objectives:
- Students need to be able to use scaled values and convert units to find the actual scaled area.
- Some important mathematical concepts include interpreting and applying scaled values and converting units.
- The different kinds of technology used include Google Earth, GeoGebra, and public websites like Wikipedia and Wolfram Alpha.

Discussion: This task asked to find the area of Lake Erie and there were three different proposed solutions provided. The first solution involved a cutout of the lake what was scaled down. The cutout was then weighed and converted to cm^2 using the density of the paper of the cutout. Then using some conversion, the answer was found to be about 24,000. The second solution used the GeoGebra features ands the tools. The third solution involved looking up the surface area from a handful of different sources. In the science classes, like physics and chemistry, a lot of the different strategies used here are implemented often. For instance, weighing objects and using the density to find the area or volume is commonly used in chemistry. Converting from units of smaller values to units of larger value or applying a scaled ratio are concepts commonly used in physics.

Anticipated Student Difficulties: One obstacle for students might be the conversion values with smaller units to values with larger units based on the scale of the map. In the first proposed solution, you are required to convert from cm^2 to m^2. Students need to know that 1 cm^2 must be divided by 100^2 m^2. Another example of converting that could be troublesome is seen in the first proposed solution where you are asked to convert the weight of the paper to the area using the density of the paper.

Extensions: Naturally, I think students would begin to apply the skills they used and the understandings of this problem in future contexts. For example, converting to and from different units and applying a scale are pretty common mathematical and scientific practices. They might apply the skill learned by using the density of the paper to find the surface area, which is something that students likely wouldn't otherwise think of on their own.

What you learned – What's new to you: The GeoGebra application is something that I am not familiar with, so it was interesting acquainting myself with it. Also, I've never thought of using the weight of a cutout piece of paper to find the surface area of a large-scale object like a lake.

Rachel Hoard

Task 1.7: Zebra Crossing: Using Graphs, Models, and Perspective to Determine Height

Keywords
Graphs, perspective, calculus, trigonometry, distance, relation, ratios, geometry

Overall: 3/5 Not engaging (not relatable to students); too difficult with how it is currently written.
Engaging: 2/5 **Difficulty:** 4/5 **Math Content:** 5/5

Content Area: High school Calculus or higher class. With some adaptation, could be made into a trigonometry problem, or could simplify to algebra level as well.

Objectives:
- Be able to use technology to simplify a problem to come up with a possible solution.
- Be able to use ratios to determine height of an object.
- Be able to use graphs to represent a situation.

Discussion: This model seemed a little too abstract to me. It didn't capture my interest and it even seemed hard to follow at points. The GeoGebra use didn't seem clear to me. It seemed like the proposed solutions were just spitting GeoGebra actions at me, without explaining what they truly do. After reading the modeling task several times, I think I finally understood what was happening in GeoGebra speak. I think that this might be an effective tool to use in the classroom, but only if my students were able to have a short introduction course to GeoGebra and what all the tools can do. Overall this is not a modeling task that I would generally use with my students. I feel like the objectives of this task could be completed in a better way other than this problem.

Anticipated Student Difficulties: Students may be confused as to where to even start with this, unless given the original graph and some tips on starting the task. If have not previously worked in GeoGebra, students might be extremely confused on how to work with the different tools that it offers. I think this will be difficult for students to piece together just given the picture and the question.

Alternate Uses Mathematically:
Potentially, you could use something like this in the real world to find the height of someone who robbed a store/bank/etc.
This could also teach students that sometimes a simplified picture of a difficult task can be very beneficial.
This can teach students how ratios work and how you can determine the heights (etc) of other objects based on the height (etc) of an object that you already know.

Ideas about Implementation/Experiment:
An experiment that might be interesting to students would be if the students were to recreate this

task to see how you can truly determine someone's height. They could do the task the same way, find a fellow student's height, and determine how much error there was in this task.

Matt Wisniewski

Task 1.7: Zebra Crossing

Keywords

Distance, relation, geometry, ratios, graphs, perspective, calculus, trigonometry

Description

The task deals with finding various distances in a photograph of many strips of a crosswalk in relation to the unknown location of the camera that has taken the picture.

Overall: 4

Engaging: 4 **Difficulty:** 4 **Math Content:** 4

Objectives:

- Geometric representation of the real life picture
- Relating ratios to help solve the problem
- Ratio of distances and scale factors were important in this task
- Turning the information given and the picture given into a model/ geometric representation that aids in the completion of the task
- Some sort of graphing and computing technology (Geogebra)

Discussion: This task involves an initial step that requires some problem solving and logical thinking. The first major step that is needed is to take the information given in the pre-task description and relate it to the picture that is connected to the task. This is similar to the tasks we have had in our 400-level TE classes, as the stress of cognitive demandingness often requires transferring information to pictorial or geometric representations. The other key element of this task has to deal with breaking down the model and using knowledge of triangles and ratios in order to piece together enough connections to start solving the question posed by the task. Lastly unless instructed to, I do not believe my first choice for working through this task would have been Geogebra. We have always been directed down the path of paper and pencil.

Anticipated Student Difficulties:

The solution 1, introduces an "error" possibility when the geometric representation assumes that we are looking at a perpendicular angle to the photo being shot. This could cause problems for students if they do not understand this or take this into account.

Students may encounter misleading ideas when they try to assume anything about the position of the camera. There is the possibility for many assumptions that can misdirect the initial thought processes.

Ideas about Implementation: I think this problem would work well with a topic of similar triangles. I would get students trying to understand the relationships between the different angles and lengths depicted in the model.

Chapter 2

Linear Models

Megan Campbell

Task 2.1: Women faster than men? Modeling olympic data to find solutions to linear equations.

Keywords:

Intersection, linear equations, line of best fit, rate

Grade Level: Algebra 1, solutions to linear equations

Overall: 4 - This was an engaging and advanced task but also was accessible to students.
Engaging: 5 **Difficulty:** 4 **Math Content:** 3

Objectives:

- Model shows big picture over time.
- Learn some limitations of modeling on real world applications
- Visually see what a solution to linear equations looks like.
- Technology can be utilized or omitted.

Discussion: This task gives real data of Olympic gold medalists' times (men against women) in a 200 meter sprint. Students graph this task and then have to find a line of best fit (for each gender) and find where they intersect. The men's times are declining at a slower rate than women's. This means at some point in time the lines will intersect and students can find the solution to the lines.

Anticipated Student Difficulties: Something that stood out is that depending on the lines that the students created there may or may not be a feasible solution to the lines. Meaning they may intersect at a negative point in time rather than in the future as planned. I would let students explore this as a class rather than try to avoid this from occurring. While students are working by probing them through this situation will enrich what they get out of the activity.

Alternate Uses Mathematically: Besides the connection to the solution of linear lines, this task can be related to the discussion on the validity of models and identifying how new information effects models. When students find their solution to the olympic running times they might not be humanly practical but based on their model they seem accurate. Or if students are given new information their line of best fit my change enough that now their linear lines won't cross.

Extension: I have seen a task like this where the students chose their own two points to create a best fit line by hand and then calculated the accuracy of that line by the following process. Take each original point and subtract it from the y-value on the best fit line that corresponds to that same x-value (finding the distance from the original point to the line). Then square the distance, this is then repeated for each point and summed. The larger this number is the less accurate. That could be a way to connect this linear task to statistics.

Technology: Technology can be eliminated. Rather than finding the intersection using technology students could try to find the solution by hand. This decision should depend on what point this task is placed in a linear unit, is it an introduction to linear solutions or a project at the end of the unit.

Rachel Hoard

Task 2.2: Taxi Companies: Modeling linear and multivariable equations

Keywords
Intersection, equations, linear equations, rates, multivariable equations

Content Level: Algebra 1: Linear equations; Algebra 2: two variable equations in 3D

Overall: 4.5/5 Different parts can be used for multiple classes. High math content and relatable to students. Not extremely difficult.
Engaging: 4.5/5 **Difficulty:** 3/5 **Math Content:** 5/5

Objectives:
- Be able to find where two equations intersect.
- Be able to use technology to see algebraic solutions in 2D and 3D.

Discussion: This model is a great way to introduce linear equations to algebra students. The analogy of a Taxi Company could be relatable to many students, especially if used in an Urban school. Students could use the information that they learn to properly select a taxi company for either themselves to use, or even for their family to use. This gives a practical application to the problem, which makes the problem more effective in a classroom. There are two similar parts to this task, one using just linear equations, and one using an equation with two variables in it. The first part with just the linear equation could be used with Algebra 1 students. The second part with a two-variable equation could be used with Algebra 2 students. Though it may be difficult at first for students to think of these equations in 3D, it would be a thought provoking task that would be beneficial for the students.

Anticipated Student Difficulties: Students may find difficulty in visualizing what a line looks like graphed in 3D, even with help of the software. This could be addressed by actually creating a "graph" in real life with the students. You could set up the different axis' and have students hold a string and move it as the line of the equation.

Alternate Uses Mathematically:
This would be very beneficial for students who live in an area where they may use taxi's regularly. They could use what they learn in order to select the cheapest taxi company for the trip they are taking.
This can also be transitioned to many other linear representations to fit the needs of the students.

Ideas about Implementation/Experiment: Algebra 1 Classroom Idea: You could create different groups of students and have them work amongst themselves to find out how much a trip would be for several certain amounts of miles. Then the students could compare with the other groups to see which groups "taxi service" would be the best to use for the trip distances.

Ryan Gronwick

Task 2.3: Can you take the collected data based on crime development to predict the number of crimes in the future?

Keywords
Data trends, linear equations, quadratic equations, exponential equations

Content Area: Algebra
Overall: 3 stars – average of the three ratings below
Engaging: 2 stars – this task could have been better if the students did the data collecting
Difficulty: 3 stars – creating a line of fit, interpreting and applying it can be difficult
Math Content: 4 stars – some key mathematical ideas are brought out in this task

Objectives:
- Students need to be able to find a line of best fit from given data and use the line to find future values.
- Important mathematical concepts include data trends (linear, quadratics, exponential, etc.), interpreting the trends and finding the error in the fit line created.
- This task can be done by hand, but a computer program like GeoGebra is used for more accuracy.

Discussion: The goal of this task was to find a model of the provided data about crime trends and to use the data to find the amount of crimes committed in a future year based on your line of fit. This task taps into students' knowledge about different types of functions that would fit different trends. Students must be able to apply their algebra and statistics knowledge to this task.

Anticipated Student Difficulties: Depending on the age of the students, I don't think it would be too challenging for them to input the data to create a best-fit equation. However, I think students will run into trouble when asked to determine if their model is reliable. Their predictions for the future based on their models might not be totally accurate because the actual data trend can be unpredictable. The actual data for the years the students are asked to predict could be significantly higher or significantly lower based on a variety of factors. This could be hard for students to recognize.

Another anticipated difficulty was recognizing the change in trends. About halfway through the graph of the data, the trend goes from exponential to linear. In order to find the future values, you would use the second half of the data to stay consistent. If students don't recognize this they would have a totally different line of fit.

Alternate Uses Mathematically: This problem could be used in a variety of different ways that include different data trends and predicting future outcomes.

What you learned: I wouldn't have recognized the change in data trend when it switches from exponential to linear.

Sakthi Shanmugasundaram

Task 2.4: Thermal Expansion

Keywords

Linear regression, thermal expansion coefficient, linear equation

Content Area: Algebra with a science focus

Overall: 4 (Even though the steps are straightforward it allows students to explore and work with math and science to discover)
Engaging: 4 (It has the students working with data points to figure out how mental can expand)
Difficulty: 2 (The steps are given to the students to work through the math behind the modeling task, but to be able to understand and explore comes from the students' knowledge)
Math Content: 3 (Works with linear regression)

Objectives:

- Students will be able to figure out the line of regression
- Students will be able to figure how what mental they are using and what the thermal expansion coefficient of the mental is (more science related)
- Students can use GeoGebra or a calculator
- The calculations can be done on paper or using GeoGebra
- Students will be able to explore using GeoGebra

Discussion: This task walks through students on how to determine a regression line for a mental experiment. The concept of the modeling task is thermal expansion, and thus there is a mental that is heated at different temperatures over 100 degree Celsius and then how much the mental expanded is noted down. Using the information students can create a regression line and calculate the thermal expansion coefficient depending on which mental they used. The data that the students collect and put together helps them lead to which mental they were working with the entire time.

Anticipated Student Difficulties: The steps that the task walks a student through is very detailed but it may be hard to follow along for a first time GeoGebra user. I think that since the steps are laid out for the students, this task may not be hard for the students. If the students collected inaccurate data it may be hard for them to figure out which kind of mental they were using.

How can we help Student Difficulties?
Make sure to give students exploring time with GeoGebra and previously have worked on easier GeoGebra uses before doing this modeling task with them
Let students know that there can be errors and talk about them before they begin the task

What did you learn: I used geo-gebra for this modeling task and it was really cool to see how graphically and mathematically the numbers turned out for how the mental expanded. I think it would help students to see how the change occurs when temperature increases.

Sam Barra

Task 2.5: Exploring Stock Options

Keywords

Piecewise functions, inequalities, domain and range

Content Level: Finance/Accounting

Overall: 2 stars
Engaging: 1- they task is extremely wordy and personally I would be completely unengaged if given the large amount of financial vocabulary
Difficulty: 3- the difficulty lies in the initial decoding of the problem. If you can figure out the wording, then the actual GeoGebra part is not extremely difficult.
Math Content: 3- does ask students to come up with the piecewise equations and functions

Objectives:

- Discussing inequalities and domains
- Creating equations and inputting them in GeoGebra
- Piecewise functions in GeoGebra

Discussion: This task is related to buying and selling stocks. We had less complicated tasks that were similar to this when I took my economics class in high school. Personally I feel like a problem with this much content would be more useful for a class like that or more specific finance class.

Anticipated Student Difficulties: The overall vocabulary in the task may be difficult for some students. In addition to all the involved math, this problem gets extremely specific with the types of stocks and financial meanings. I think that a lot of this context is completely unnecessary if this is a typical mathematics class and the idea of representing piecewise functions in GeoGebra could be displayed with a much less wordy problem.

Extensions: Would be a useful problem in a finance class to incorporate math and stocks together.

Experiment: You could have students watch stocks online and act like they are actually taking place in the stock exchange. They would theoretically buy and sell stocks and record their data for a certain number of days and then could graph their data in GeoGebra to see how it would compare to the functions that were graphed earlier.

What you learned: http://www.investopedia.com/simulator/ online simulator to buy stocks and bonds and use for the experiment above.

Task 2.6: Flying Foxes

Keywords
Error, error bars, fit, best fit, equations

Overall: 4
This task showed the differences of fitting different kinds of models and how error bars appear on graphs. It require basic GeoGebra that made the task more based on discovery and not computer knowledge. The question was also open ended so that students could try other fits to see how it would appear.
Engaging: 4
Students get a say in what kinds of fits they would like to do. There is not really a wrong answer either which leads for them to think more outside of the box. Students can show how they figured out the highest and lowest values possible according to their fit to other students easily and show understanding.
Difficulty: 2
This task has very little difficulty if any. The difficulty comes from thinking of different fits and knowing how to calculate the error of each. This also does not allow much connection beyond this such that once you find the equations you have to look at where the lines are at 2020.
Math Content: 3
This shows the differences in equations which can be very helpful for students trying to picture them all in their head. It also shows how two points are very vague and more information is needed.

Objectives:
- Find out which equation is most practical for this assignment.
- See the changes in errors with different equations

Anticipated Student Difficulties:
The only difficulty I could find students having are thinking of different equations to fit with the points. Also seeing that the error bars play a role.

Extensions:
The differences in the equations at the beginning compared to farther down the road.
What would happen if we added more points? Would this force us to choose only one equation for best fit?

Chapter 3

Nonlinear Empirical Models I

Trevor DeBono

Task 3.1: Galaxy Rotation

Keywords
Fit, function, minimum, regression, rotational velocity, distance, extremum, critical points

Overall: 4
Engaging: 4 **Difficulty:** 2 **Math Content:** 4

Objectives:
- Students will be able to fit a function to a list of data points
- Students will use regression to determine a function that would accurately find the minimum value
- Students will use Geogebra to model these functions and interpret the data

Discussion: This task looks at the theoretical rotational velocities of a circular galaxy at given distances. The purpose is to find the distance at which the minimum velocity will be recorded. Multiple data points are given to create a function and some interpretation is required.

Anticipated Student Difficulties: Some difficult may be realizing you can create functions from some, not all, of the data points included. As a result, different conclusions can be met. Additionally, the units included may be new to students and can be hard for them to interpret.

Ideas about Implementation: Included in a unit about finding extremum of graphs using critical points of a graph.

Yaming Liu

Task 3.2: Olympic pole vaulting: Construct different models based on the existing values, calculate the height in 2020 and explain which model is better

Keywords
Linear equation, linear fit, logarithm, logarithmic fit, logistic, nonlinear model, best fit

Overall: 3
Engaging: 3 **Difficulty:** 3 **Math Content:** 2

Objectives:

- Make different models based on the given values and explain which one is better.
- Know the linear model formula ($y = kx + b$)
- Know the logarithm formula ($y = a*\ln(x) + b$)
- Apply linear model and nonlinear model (logarithmic model)
- Know how to use GeoGebra (make tables, draw graphs and apply the fitness line)
- Know to change the a, b, c values to make a better logarithm model

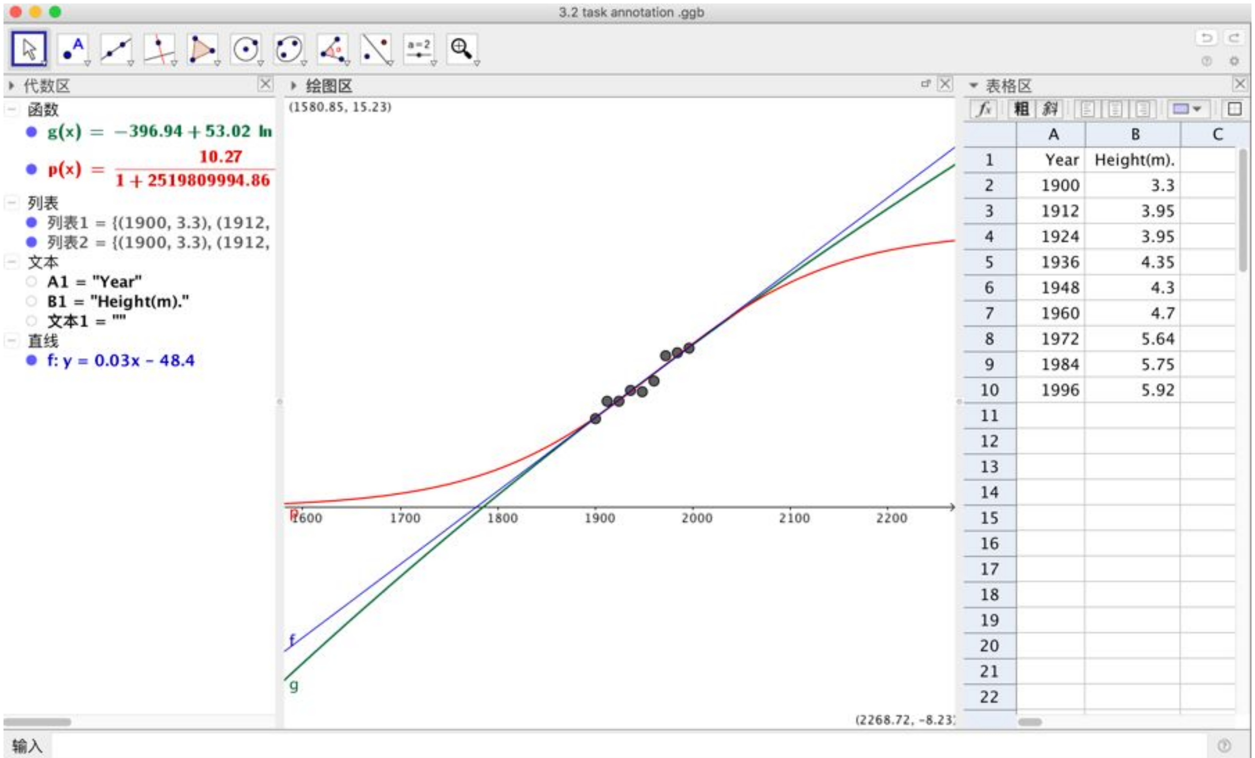
Discussion: We could see the linear line passed through multiple points in figure 3.6 and the logarithm line did not pass through any points in figure 3.8. But the case showed the logarithmic model fitted better than the linear model. Why this is the case? (Because we don't expect the value will increase all the time, the future points should go close to horizontal.)

Anticipated Student Difficulties: There are many models could solve this problem. But it is hard for students to know and find a logarithm model works better than other models. Also, change a, b, c values to get a better logarithm function is very hard for students.

Alternate Uses Mathematically: Except the linear model and logarithm model, I think the logistic model is also a great choice to fit the points since we should not get the graph pass through the negative height.
In the pole vaulting case, there are other problems could be discussed. Take an instance, based on some given values, we could figure out the velocity of the athlete when he gets to the highest point. And then we could solve the distance he could travel. Then we transfer this kind of problem to the physic aspect.

Ideas about Implementation: There are other factors could make the different height the athlete could get, such as the material of the pole, the malleability of the pole (soft or hard), the shape of the pole at both ends (ball or other) and so on. And then, we could make comparison experiments to show what kind of the combination of the external factors is the best.

Experiment:



What you learned: I learned that the logarithm model (curved) usually works well with athletic records.

Abigail Johnson

Task 3.3 Kepler's Third Law and the Orbits of Planets

Keywords

Planets, orbit, time, kepler's third law, Titus-Bode's law, linear function, best fit, sliders, power function, approximation

Description

In this task, students are provided with data about planets' distances from the sun and the time that it takes each planet to complete a full orbit. From this data, students are expected to use GeoGebra to determine the relationship between distance and the length of an orbit.

Overall: 3

Engaging: 3 **Difficulty:** 3 **Math Content:** 3

Objectives:

- Students will be able to use sliders to approximate a line of best fit for a set of data
- Students will be able to use several of GeoGebra's "fit" tools in order to generate a line of best fit and a best power function

Discussion: This task is an excellent introduction to the functions of GeoGebra and their applications in describing mathematical models. The task explains to students how to input data into GeoGebra and represent it on a coordinate axis, then guides students through the process of exploring a line of best fit through trial and error using sliders. It then takes students into an exploration of the different ways in which they can fit points to a line using the "fit" functions, as well as how they can fit data to a power function. This is a fun exploration of the tools featured in GeoGebra, framed in the context of selecting a type of function most suitable to model the relationship between planets.

Anticipated Student Difficulties: The focus on finding a line of best fit, both through approximation using sliders and several of GeoGebra's fit tools, may lead students to believe that a linear approximation is appropriate for this data set even though it is very clearly not the best approximation to use.

Extensions: This task could be extended by having students manipulate the data in an attempt to find a linear relationship. i.e., seeing how the relationship and appropriate functions change when distance is squared, orbit time is squared, either is halved, multiplied by the other, etc.

Alternate Uses Mathematically: This could also be used to introduce the idea of error and using the sums of squared errors to determine which model is the best representation of a set of data.

What you learned: I learned that the length of a year on a planet is related to its distance from the sun by a power function, I never knew about that relationship before. I also learned that lines can be fit in GeoGebra using either FitLine (my go-to method) or FitPoly with a degree of one.

Task 3.4 Density

Keywords

Functions, best fit, error, error analysis, temperature, volume, density, maximum

Content Area: Algebra 1 or 2, potentially Calculus

Description:

Try to find two different functions that fit these data points. Also, carry out some form of suitable error analysis. Determine the best data fit, that is, which one of your functions fits the data points best, as you see it. Also, determine at what temperature water has its smallest volume, meaning its largest density. Comment on the following statement: "Water has its maximal density at the point of freezing."
[Before the problem there is a chart with 17 paired numbers.]

Overall: 3
This problem allows for the possibility that all students be semi-successful with little-to-no guidance from the teacher. The solution can be determined from graphing calculators, GeoGebra, or by hand [much more difficult], so the problem can be presented in any classroom.
Engaging: 3
The problem presents graphing points in an interesting manner. Students who normally find no interest in graphing now have an engaging problem because there is a reason behind doing so.
Difficulty: 3
Since the problem requires that students fit the points in two lines, they must generate a solution that most students have not heard/worked with [quadric]. Students also have to find errors of the projected lines. [Same for below]
Math Content: 3

Technology Requirements: Computers equipped with GeoGebra, students with the capability to work with and manipulate charts/graphs in GeoGebra.

Objectives:

- Students will be able plot points on a graph and fit two different lines.
- Students will be able to find the errors between their fitted lines and the plotted points.
- Students will be able to use the fitted information to find general information that corresponds with the data given. (Smallest volume at which temperature, largest volume at which temperature, etc.)

Anticipated Student Difficulties: Students could find difficulty in figuring out which two types of lines the solution method suggests. Most students have not heard or worked with quadric functions and what they look like graphed. A way to accommodate for this problem would be to introduce quadric functions, what they look like graphed, and the formulas the lesson before so that it is "fresh" in their minds.
Another issue that might not be as problematic, but could still cause issues is asking students to realize on their own that subtracting "999.5" from each value to let the programing work in GeoGebra. The best way to help students overcome this difficulty is to point them to similarities between each output value.

Task 3.5: Different Ways to Find Parameters of Logistic Functions

Keywords

Spreadsheets, rate of change, functions, carrying capacity, growth rate, linearization, linear function, logistic function, derivative rules

Description

Given data of growth of yeast cells, model the relationship between time and population, and determine carrying capacity.

Objective:

- Increase familiarity with calculations in spreadsheets
- Exploring and understanding relationships between rate of change and original function graphically
- Exploring different ways of calculating carrying capacity and growth rate parameters, using various rate of change graphs
- Exploring concept of linearization
- (For calculus students) Verifying derivative rules directly through data

Exploration/Discussion: Initially, fit a logistic function to data, note equation has form $P(t) = M/(1 + Ae^{-kt})$. Note values of parameters k and M. This is one way of obtaining parameters. However, other ways are to work directly from the data:

- Calculate $\Delta P/\Delta t$, $\Delta^2 P/\Delta t^2$, $\Delta P/(\Delta t \cdot P)$ from the data. Then plot $\Delta P/\Delta t$ vs t, $\Delta P/\Delta t$ vs P, $\Delta P/(\Delta t \cdot P)$ vs. P, $\Delta^2 P/\Delta t^2$ vs. t, $\Delta^2 P/\Delta t^2$ vs P. What do you notice in each of the graphs? Where are the zeroes? What is the slope (if linear) and/or how can we describe the rate of change in each graph? How does the zeroes and slope relate to k and M?
- Can we calculate k and M from each graph? If so, how do each of those values compare with the k and M from the original fitted logistic function?
- Which graph is the easiest to work with? What does this graph tell you about relationship of population to rate of change?

Anticipated Student Difficulties:

Why learn different ways to do something if we can just fit a regression and be done?: In real life, we will encounter data in which we do not immediately know the model. Looking at various rate of change graphs could provide additional insight into the behavior of the data.

Why are the values of k and M calculated from each graph slightly different?: Note when we calculated $\Delta P/\Delta t$ and $\Delta^2 P/\Delta t^2$, we had a choice between using left-hand difference, right-hand difference, and symmetric difference. Try exploring how k and M varies with calculated data using each difference!

(Calculus students): What does any of this have to do with the derivative? Differential equation has the form $dP/dt = kP(M-P)$. Second derivative is $d^2P/dt^2 = kP(kM-2kP)(M-P)$. We could fit our $\Delta P/\Delta t$ vs P and $\Delta^2 P/\Delta t^2$ vs P graphs, obtain values of k and M, and compare them to parameters from the original fitted logistic curve. According to derivative rules, those parameters ought to be about the same!

Bowei Zhao
Cayla Pearsall
Matt Champagne
Matt Wisniewski
Ryan Gronwick
Stephanie Young

Task 3.6: Finding a fit equation to represent the behavior of liquid cooling

Keywords
Best fit, function, law of cooling, exponential function, temperature, sequences, geometric

Overall: 4 stars (average of the three below)
Engaging: 4 stars (Students can actually run an experiment with this specific sketch.)
Difficulty: 3 stars (It is pretty average in terms of difficulty. You just have to record data, plot it, and be able to interpret a best fit line.)
Math Content: 5 stars (This is a really great way to engage students in mathematica ideas about exponentials. It might require students to reason as to why some fits are better than others.)

Objectives:

- Students will try to identify which type of function provides the best fit to the data. Thus, a knowledge of the different types of functions is needed or can be learned through this.
- Students will need to know laws of cooling and the exponential functions that go with cooling and temperatures.
- Students will need a program that can plot points and find a line of best fit.
- (Alternative use) Students can explore the connection between geometric and arithmetic sequences through exponential functions.

Discussion: This sketch works with the cooling of a hot drink at room temperature. It takes a handful of data points and plots them to a graph and then a line of best fit is created for the data. The sketch is a real-world application of an exponential graph that would require students to reason with what factors might affect the cooling data trend.

Anticipated Student Difficulties: In the first proposed solution, it recommends that you remove the first six points to take into account for the cup "gobbling" up some heat initially. Students will likely not think to account for this (I, Ryan, would not have). Also, the first few points may yield a temperature ratio which deviates from the theoretical (see Alternative use). In either case, we would explore questions about how the experimental design, as well as other experimental/environmental factors, might affect our results.

What you learned: I thought it was neat when the first 6 points were removed. Recognizing that the cup needing to be heated up first shows the variety of variables that go into real-world problems like this.

Alternate Uses Mathematically:

Let $T(t)$ = temperature at time t, T_0 = initial temperature, T_r = room temperature.
Newton's Law of Cooling: $T(t) - T_r = (T_0 - T_r)e^{-kt}$.
Define $\bar{T}(t) = \frac{T(t)-T_r}{T_0-T_r}$ and constant $A = e^{-k}$. Then law of cooling expresses as $\bar{T}(t) = A^t$.
Assume data is taken at equal time increments $t_{n+1} - t_n = \Delta t$.
Observe $\{\bar{T}(t_n)\}$ is a geometric sequence with ratio $A^{\Delta t}$, paired with arithmetic sequence $\{t_n\}$ with difference Δt. (If $t_0 = 0$, then $\Delta t = t_1$)

$\boldsymbol{t_n}$	$t_0 = 0$	t_1	$t_2 = t_1 + t_1$	$t_3 = t_1 + t_1 + t_1$	*and*
$\boldsymbol{\bar{T}(t_n)}$	$\bar{T}(t_0) = 1$	$\bar{T}(t_1) = A^{t_1}$	$\bar{T}(t_2) = A^{t_1}A^{t_1}$	$\bar{T}(t_3) = A^{t_1}A^{t_1}A^{t_1}$	*so on...*

This relationship is verifiable directly in the heating and cooling data.

Task 3.7: Population of Ireland

Keywords
Function, best fit, population, predictions, questioning

Description
Given the population of Ireland 1672 to 1966 students are asked to formulate questions based on the data in the chart then fit a function to the data if possible and answer their questions.

Overall: 4
Engaging: 4
Population is something that students can relate to because the problem can be made into a visual representation or modified to represent the change of population in the classroom. Students also get to create their own problems and answer them so they take ownership of their learning.
Difficulty: 4
I think the use of geogebra to decide which graph fits could be a difficult task. Students could also have differing opinions of which graph fits the best and it could be difficult to come to a conclusion.
Math Content: 3
Fitting a graph is an important concept in math however it is not very difficult to do using a computer program. The problem does ask for a function to be created from the function of best fit so students also could be asked to explain the parts of the function.

Objectives:
- Graphing functions from given data
- See patterns on graphs
- Comprehend functions
- Understand parts of a function
- Relate functions
- Relating mathematics to other disciplines
- Critical thinking of data
- Creating mathematical questions
- Geogebra

Anticipated Student Difficulties:
It could be difficult for students to decide which function fits best and therefore create a debate in the classroom however this could also been seen as a positive.
The given solution divides the data into 2 parts; before and after emigration, to create 2 separate functions of best fit. I think that getting students to see the two separate function would be difficult and take some scaffolding however, once noticed there is a lot to discuss.

Brent Nutt

Task 3.8: The Rule of 72

Keywords

Algebra, rational function, exponential function, log function, comparison, money, interest rate, doubling time

Description

This task mentions a common formula to calculate how long it would take a certain amount of money to double, and asks the student to compare it with a more correct formula, to see how much error there is in the more simple formula, for different input values

Overall: 4 - This task is relatively straight-forward with the questions that it asks, but it still requires a decent amount of effort, math, and exploration to be able to find the answers.
Engaging: 4 **Difficulty:** 4 **Math Content:** 4

Objectives:

- Algebra - Uses the ideas of rational functions (like y = 1/x) and logarithms.
- GeoGebra or a graphing calculator would be very helpful.

Discussion: This task mentions the common "rule of thumb," that you can calculate the amount of time it takes to double a certain amount of money in a bank account, by dividing 72 by the interest rate. There is a more accurate way to calculate this amount of time, by using exponential functions (and log functions). So, this task asks the student to compare those two ways of calculating the doubling time of the money, and to try to find out when the "rule of 72" method is close enough to the other method.

Anticipated Student Difficulties: Students might have trouble remembering how to use an exponential function to model the gradual growth of an initial amount of money. So, they might just need some initial reminders about that.

Other information: I think this task is pretty suitable for high school students the way it is right now. The description of the task is pretty concise, and the questions that the task asks are relatively clear. I also think this task gives the students a good opportunity to explore the math for themselves, to learn more about rational, exponential, and log functions, but it is also straight-forward enough that it probably won't overwork or overwhelm students as much as a more difficult task might. I also like the way that this task relates well to an actual real-world situation.

Cayla Pearsall

Task 3.9: Fish Growth Rates

Keywords
Rates, exponential growth, variables

Content Area: Algebra , Growth Functions/ Quadratics

Overall: 3.66 / 5
Engaging: 2/5
Some students will not find fish weights engaging.
Difficulty: 5/5
This problem is quite difficult and time consuming to do in GeoGebra
Math Content: 4/5
This problem contains exponential growth.

Objectives:
- Exponential Growth
- Understanding how the variable of a growth equation relate with one another.
- GeoGebra was the only tech option used.

Discussion: I feel that this problem is a good problem for deepening a student's understanding of the various variables of an exponential growth equation.

Anticipated Student Difficulties: I found it very hard following along with the proposed solution because it does not exactly say where to input parameters at, for the first graph. Also just following along and making the graphs unless you remember what each slider stands for. Not quite sure what the variable a stands for in the equation and as a slider.

Ideas about Implementation: Could change the object/ animal that the growth rate applies to, so that it relates to the students.

Chapter 4

Nonlinear Empirical Models II

Amber Smith
Megan Campbell
Sakthi Shanmugasundaram
Brent Nutt
Karlie Goretski
Trevor DeBono

Task 4.1: Cooling Activity 2

Keywords
Models, cooling, compare and contrast, error, law of cooling, functions, exponential function, polynomial, fourth degree polynomial, sixth degree polynomial

Content Level: Algebra 2 or Calculus

Description
Given a subject matter heated to 100deg C and specific data, construct two models of how the subject cools. Find the errors of the models and discuss the similarities/differences between them. Additionally find the time at which the subject measures 31 deg C.

Overall: 3
This problem could be potentially difficult to carry out in a course. A lot of small errors can be made which could make it difficult to keep everyone together during a lesson. This could also give students the opportunity to use logical reasoning to figure out where they went wrong. Also it may pose difficulties if they do not have knowledge of Newton's Law of Cooling which is often not found in the math curriculum - especially if you add the GeoGebra application on top.
Engaging: 2 **Difficulty:** 4 **Math Content:** 3

Objectives:
- Students will be able to determine two relationships given one set of information.
- Students will determine the errors within their model.
- Students can compare and contrast properties of different graphical relationships.
- Students will be familiarized with Newton's Law of Cooling in a math perspective.
- Students will get practice working with and manipulating various different types of functions (like exponentials and fourth-degree or sixth-degree polynomials).

Technology Requirements: This task heavily relies on GeoGebra, and students' ability to manipulate information within the program. The problem gives step by step solution methods that students could potentially follow but some seem confusing and may need the teacher to revise so that students understand.

Anticipated Student Difficulties: Students might find difficulties in determining errors in the models they create. A way to get around this would be to go over a problem before and give the class a task that requires you to find the errors. After you could scaffold and the use whole class discussion to find the errors in that specific example that could be applied generally.

Another possible area that could create challenges for students in the classroom could be using Newton’s Law of Cooling. As mentioned before, this is not typically found in math curriculum and students likely wouldn’t consider it as a possible solution method unless it was discussed directly. One way to combat this would be to introduce Newton’s Law of Cooling a day or so before this task so that it is fresh in students’ minds, or even at the beginning of a unit to connect it throughout. Another way would be to use whole class discussion and assist the students by giving them this as a first model option. Doing this might lower the cognitive demand of the task, but then students would be on the same page, and could compare this model to a different one that they come up with, instead of two that might not connect to the Newton’s law of Cooling model.

Karlie Goretski

Task 4.2: Body Surface Area

Keywords
Functions, units, body mass index, surface area, empirical formulas, data analysis

Description
"Students will use power functions to represent the body surface area of the relationship between body size and bodily functions."

Overall: 4- This task allows students to use numerous different formulas and methods including GeoGebra and Wikipedia to determine someone's body surface area.
Engaging: 4- This task allows students to use a real- life mathematical example of measuring body surface area and compare units of height, weight, and density.
Difficulty: 4- This task can be tricky to read and understand at first. It took me several times of reading through to be able to understand the material. Also, students need to know how to use GeoGebra proficiently in order to solve for someone's body surface area.
Math Content: 3- With the methods given, students have to know what the units for weight, height, and density are and thus determine how they work out. They also have to know the definitions of each of these terms, as well as body surface area and body mass index. Then, they have to use formulas and data and plug it into spreadsheets in order to compare the solutions received.

Objectives:

- Students can use empirical formulas to determine someone's body surface area.
- Students will conduct unit analysis tests to determine the shape of a function.
- Students will use corresponding units for weight, height, density, body surface area, and body mass index.
- GeoGebra is the technology required.

Discussion: This task allows students to use power functions to relate body sizes and bodily functions to determine body surface areas. With this, students must know which units are required and thus have to perform a unit analysis. By using a body surface area formula found on Wikipedia, students can determine the body surface area of sixteen individuals and compare them. Then, students can use spreadsheets on GeoGebra to enter the data for sixteen individuals' weight, body mass index, and body surface area in order to solve for the height of each individual.

Anticipated Student Difficulties: It will likely take students a while to grasp the purpose of this task and they will likely have to read the task several times to fully understand it. Students may have difficulties performing unit analysis tests if they do not know the correct units for weight, height, body mass index, and body surface area. If students are not familiar and comfortable working with GeoGebra, they are likely to encounter issues entering the data into the spreadsheets, finding the sum of squared errors, or figuring out how to round to ten decimal places.

Matthew Champagne

Task 4.3: Warm Blooded Animals: Finding head production

Keywords
Graphing, logarithmic function, heat output, best fit

Overall: 3
Engaging: 3-4
While this task boils down to graphing and understanding log functions, it has an added context of working with them to calculate the heat output of an animal. I do not think this will either be very engaging or not at all for a student, I do not think there will be a middle. This is why I gave this task a 3 or 4 star engaging
Difficulty: 3
The math itself is not that difficult, but the concept of coming up with an equation or figuring out how to use geogebra could be challenging for some students.
Math Content: 3
As the task stands right now, math content is a little lacking. The task is simply to calculate the predicted heat output of a random size animal using geogebra or a function.

Objectives:
- Students will be able to analyze data and fit a model into geogebra
- Students will be able to understand what type of function is represented by a set of data
- Geogebra or a calculator will be necessary for this problem.

Discussion: I think that students could easily check out of this activity. If they are not interested in animals they will not engage that way, and if they do not understand the math concepts they might also get frustrated and check out. If these two problems can be avoided then I think this could be a good activity.

Anticipated Student Difficulties: I think the students will struggle in two areas. First I think students would struggle with using geogebra and getting their results that way. Second, I think that students will also struggle with determining what type of function is represented by the data set.

Ideas about Implementation/experiment: I think the best way to implement this so that all students are engaged, or at least interested, you could have all of the students weigh any animals they have and bring the data to class. This way it will make the activity real for the students and it might keep them engaged and interested for a longer period of time.
After doing this activity you could then measure the heat output for one of the animals (or heat output for a similarly sized animal if getting this data is unreasonable) and see how closely the students were able to predict heat output. This would turn it into a game of sorts and create interest in the students.

Matt Wisniewski

Task 4.4: Control of Insect Pests

Keywords
Functions, multiple representations, growth, power function, fit

Description
The task deals with the understanding of the relationship between the strength of traps and the amount of moths captured.

Overall: 2.5
Engaging: 2 **Difficulty:** 3 **Math Content:** 2

Objectives:
- Mathematics that author intends in the problem or arises in its solution.
  - Connecting multiple representations of functions (Table⇔ Graph⇔ Fit)
  - Fitting a data set with a function that represents the growth of the data
- Modeling as a practice or as a content standard that stand out
  - Connecting a set of data to a function that fits its growth model
- Technology requirements, uses or options.
  - Use of a graphing system that can graph and fit a set of data

Discussion: This task does not seem to contain much cognitive demand. The task is very straightforward with its directions and almost lays out the steps to take to accomplish the task too easily. The amount of critical thinking and the ability to find multiple solutions is really lacking throughout the task. As a positive, the task does provide an opportunity for students to explore fitting a function to a set of data using GeoGebra. One thing that I think could improve the task would be to present the task with more openly ended questions. Such as "explain what amount of strength in a trap will have the largest outcome of moth collection" or something along these lines.

Anticipated Student Difficulties: Something that may cause issues for students is that the task involves fitting a data set. Knowing from personal experience through our class this idea of fitting the "correct" function to a set and how to do it through GeoGebra may cause issues.

Ideas about Implementation: I think this would be a good application used during a unit on power functions. It would possibly be helpful to explore and engage students with a technological application. I think as of more detailed implementation it would fit at the end of a power function unit for further exploration.

What you learned: I think the biggest takeaway from this task is exploration of GeoGebra and using the fit functions while understanding the idea of truly fitting the model with an accurate function.

Task 4.5: Selling Magazines for Christmas

Keywords
Model, Sales, Piecewise functions, Composition functions, Algebra,

Description
Kids make money by selling magazines then the money they make that money turns into points and they can use it on premiums or get money back. Students need to find the models corresponding to the data and use them to make decisions about how to use the points awarded to get an iPhone 5.

Content Level: Algebra 2

Overall: 5 stars- Good review to cover different types of functions and study the property of composition. Situation can engage most students and it's a great real life application.
Engaging: 5 stars **Difficulty**: 5 stars **Math Content:** 5 stars

Objectives:
- Find best model for data.
- Understand the composition of different functions to create a new function.
- Find the limitations and errors in model created
- Use model to find the most beneficial situation and explain why.
- Use GeoGebra to analyze the data and build models

Discussion: The students need an understanding of functions and compositions before attempting this problem. Students have to use model to make decisions based on mathematical reasoning.

Anticipated Student Difficulties: Since there is a lot of data and specific information the students might have trouble with understanding the task and organizing the information they need to solve the problem. Students might also have trouble finding the limitations of the model they created.

Extensions: Give students a different amount of money earned from sales and have them decided what decisions they would make. Have students explain why using mathematical reasoning about the models.

Ideas about Implementation: I would suggest that when the students are given the problems that they write the important information and connect the relationships. Have a discussion about what the students understood when reading the problem to eliminate misconceptions and discuss ways students are thinking of proceeding to find the solution.

Experiment: After the task, have the students do a similar scenario in real life. The students sell candy bars or cookies and with the money they earn the students receive points. Later, with the points they earn they can decide to receive premiums or money. They have to look and the relationship between the money and points and points and premiums or money and decided how they would benefit the most. Have them write a report about why they made the decisions with mathematical reasoning to back it up.

Technology: Students should be using GeoGebra in class all year if they are going to use this technology to find the model since the proposed solutions uses complicated GeoGebra tools.

What you learned: Using piecewise functions in GeoGebra.

Task 4.6: Determining the center and size of a tumor

Keywords
Geometry, circles, area, center, error, sum of squared error

Overall: 4.33 (average of the three below)
Engaging: 5 stars (tumors might be relevant to students if they know someone with cancer)
Difficulty: 3 stars (the steps are clear and the concepts are fairly easy to understand)
Math Content: 5 stars (the application of the mathematical concepts is really good!)

Objectives:

- The author intends to bring out the ideas behind the geometry of circles as it focuses on the center of a circle and the area.
- This task brings some attention to the sum of squared error in order to identify the best position of the circle.
- GeoGebra is used. Any other geometry program could be used as well. Sum of squared error could be found using a calculator.

Discussion: In this task, we are provided with six points that lie on an x-ray of a tumor. For the first proposed solution, a circle was created with sliders to move it along the x- and y-axis and a slider was created to increase/decrease the radius of the circle. By moving the sliders, a circle that best fit the points can be found. In order to increase its accuracy, an equation for the sum of the squared error was created so we can visibly see how much the circle deviates from the points. The second proposed solution, a conic was created and two lines were created to intersect at the middle. For both of solutions, the center of the circle would be the center of the tumor and the area of the circle would be the area of it. A task like this can be seen in a geometry or statistics class.

Anticipated Student Difficulties: This task is pretty straightforward, but I think students are likely to have a hard time with creating the GeoGebra file for it. It requires students to know how to create sliders and know how to manage basic GeoGebra tools. Other than that, most if not all of the concepts are relatively basic and depending on the class or grade level, students should have much of a problem with the task.

Extensions: This task could be used for anything where you are asked to find the area. You could use different shapes like squares, rectangles, triangles, or ellipses.

Megan Campbell

Task 4.7: FREE FALL

Keywords
Exponential function, data analysis, best fit, freefall, functions, validity

Content Level: Algebra learning about various functions and their validity

Description
Data is given regarding the distance fallen as a function of time for objects in free fall. More specifically talking about the number of jumpers in the air behind you when falling out of a plane, and the vertical distance fallen so far. The entire fall takes place in one second. Fit an exponential function of the form d(t)=A+Be^t to these data and make an analysis of this fit.

Overall: 4
Overall this problem wasn't too difficult and was fairly engaging since it was about objects in free fall and modeling the information. There was also multiple solutions that students could find and could support with evidence with model is better.
Engaging: 3 **Difficulty:** 2 **Math Content:** 4

Objectives:
- Students will be able to view the problem multiple ways to find the most ideal fit.
- Students will be able to support their choice of model with mathematical evidence.
- Students will have to connect multiple types of functions to one problem.
- Students will be able to explore ways of investigating the validity of a model.

Technology Requirements: This task requires GeoGebra or another dynamic software to model the given data. This could be done by hand but it would be considerably more difficult to fit multiple representations to the set of data and investigate their validity without the use of modeling software. Thus I would say technology is required to complete this task.

Implementation: I think a better way to pose this problem would be to ask students to model the data, however they see fit. Keeping it more open ended could elicit various responses by students rather than giving the task of fitting an exponential function. This could also allow students the chance to recall information from multiple function families. This would also allow a more fruitful class discussion about what model is ideal and why because more than just an exponential model will be investigated.

Anticipated Student Difficulties: By the proposed solutions in the book the modeling is set up in a way that the fit can be easily changed by just manipulating what the m(x) function is defined as. This makes it very feasible to compare a variety of function types to find an ideal function. If students do not set up their investigation this way it could be very redundant exploring the various functions. Additionally, they may lose persistence when investigating the problem and only look into one or two different functions.

Additionally I anticipate students struggling to find a way to investigate the validity of each model. The idea to look at residual diagrams might not be the first instinct for students. Furthermore, to look at the sum of squared errors might be an idea students have based on previous experience, but they might not understand the connection. Ideally the goal is to see that small errors in the residual diagram or a small sum of squared errors will be indicative of a better model and that can be used as evidence when a class discussion occurs.

Matthew Champagne

Task 4.8: Concentration

Keywords
Concentration, statistics, maximum, formula, function, models, best fit, compare and contrast, multiple representations

Content Level: Statistical methods and mathematical modeling

Description
This task gives students a table with concentration levels of a certain chemical in a reaction. The task then asked the students to find the maximum level of the chemical during the reaction.Students are given the formula $C(t) = c + a \cdot e^{(-0.47 \cdot t)} + b \cdot e^{(-0.06 \cdot t)}$ and are told that this is a decent model for the data given. The task then asked the students to explore 3 different models and see if they fit the data better (they didn't) and represent the maximum more accurately (they don't).

Overall: 3
Engaging: 3 **Difficulty:** 4 **Math content:** 3

Objectives:
- Students will be able to fit mathematical models to data provided
- students will be able to tell what model is the best fit for the data provided
- students can use geogebra to create tables, plot points, and fit models
- students will be able to connect what has changed in the model, with how it changes the graphical representation

Discussion: In this task students are initially supplied with the levels of a chemical and the corresponding data. However they are immediately provided with the model that ends up being the best fit, before they are asked to manipulate the model to see what happens. It would be better if the students were given a variation of the model and work to find the model that fits the data better.

Technology: Geogebra or a similar software would be mandatory for this type of activity. Since the students are creating several different models to fit the data, it would be incredibly tedious and serve no extra purpose to have them do this by hand.

Anticipated students difficulty: There are many typo's in the book for this particular task and took me many tries to get each model to come out the way that it did in the book. I think this will be the greatest difficulty to students because even if you are typing in exactly what you are told and have a decent understanding of what is going on, it will still come out wrong. I believe students will struggle with putting in the extra time that is required in order to figure out what is going wrong and fix it in order to see what is different between the models.

Cayla Pearsall

Task 4.9: Air current measured over the tip of an airplane, using fluid dynamics

Keywords

Current, fluid dynamics, parameters, best fit, line of best fit, error, sum of squared errors, base equation

Content Level: Algebra or physics

Overall: 2.33/5
Engaging:2/5
Some Students will not find airplane wing data very engaging or using fluid dynamic equations.
Difficulty:3/5
Students may find this difficult if they have no prior knowledge of fluid dynamics or air current
Math Content:2/5
The math is very standard for algebra "here's this data find a model".

Objectives:
- To find the parameters for the fluid dynamic equation
- Student must be able to figure out line of best fit using sum of squared errors
- Student can use GeoGebra to find the parameters

Discussion: This problem is good if your students have prior knowledge of air current and fluid dynamics. The students take data and find the best parameters for the model given.

Anticipated Student Difficulties: Student might find difficulty in creating a line of best fit for the two additional points that are not fixed. Student may also find difficulty in creating the base equation.

Task 4.10: Tides

Keywords
Tides, water level, function, error, sine, trigonometry, best fit, residuals, sum of squared errors

Content Level: Statistics, Trigonometry

Description
Given tidal data from Hall's Harbor in Nova Scotia during one session to measure the water level once an hour for two days, find a close-fitting function for this data set. Also try to make a reasonable error estimate.

Overall: 4
Engaging: 3
I am not sure if students would find this task very interesting. I do think that it could be modified so that students could measure their own waves or the waves could be measured from somewhere in which they are more familiar.
Difficulty: 4
The proposed solution was fairly simple because it simply had student plot the points and fit a sine graph to the points. From this you find the residuals, or the difference between the data and function value and plot this as well. This helps in creating a better-fit sine graph. However, after this I went a step further to find the sum of squared errors from each of the two functions to show that the second function was, in fact, a better fit. This was more challenging because the input was long and very specific and there were no directions for how to do it. I think if students were given some guidance it would be helpful for students to see proof that the second graph is closer to the data.
Math Content: 3
This task would be a great task to help students visualize the sine function and see how it can relate to the real world. It also makes it easier to understand residuals and how they can make a function more precise to the data. The fact that students can visually see that the sum of squared error is smaller for the second function drives home the fact that their graph is a better fit rather than just noticing that it goes through more data points.

Objectives:
- Students will learn more about how to work with a statistical distribution such as residuals and sum of squared errors
- Students will be able to visualize the sine function
- Students will be able to judge and change a graph to create a graph with the least amount of error

Chapter 5

Modeling with Calculus

Yaming Liu

Task 5.1: The Fish Farm II

Keywords

Linear function, quadratic function, cubic function, maximum, revenue, cost, profit

Content Level: Algebra

Description

Construct different models to calculate the prices lead to maximum profit and figure out the degree of sensitive the price changes with given conditions.

Overall: 3

Engaging: 2 **Difficulty:** 3 **Math Content:** 3

Objectives:

- Students can make a linear, quadratic, and cubic function given two points.
- Students are able to compare and analysis the graphs
- Students are able to type in the formulas directly for making the graphs in Geogebra
- Students are able to use the formulas of revenues, costs, and profit in order to find the suitable price for each fish to sell.
- Students are able to set up a slider for number of fish and price of fish to see the degree of sensitive of price changing
- Students are able to choose the numbers to plug in the functions with given situations to make a quadratic model.

Discussion: In the beginning, as I see this problem, I thought the model should be linear and quadratic functions since I have two points. However, the proposed solution 1 talks about the linear and cubic models while the quadratic model is talked in proposed solution 2. I have a hard time thinking about the reasons behind putting the linear and cubic models in one solution.

Anticipated Student Difficulties: It is hard for students to think of the cubic model since we only have two points in the beginning. In addition, in linear model, the price is 13.63 and the max profit is 13326.88. In cubic model, the price is 7.94 and the max profit is 3224.39. In quadratic model, the price is 12.792 and the max profit is 11048.557. We can see that 13.63 and 12.792 are close, which means we have similar approximations in linear model and quadratic model. However, the price for cubic mode is 7.94 and the difference of profit is about 10000 compared to other models. Therefore, I have questions on the factors lead to huge differences.

Extensions: If we are given three points, what models can we get? If we are given four points, what models can we get? …If we are given n points, what models can we get? Is there a pattern in it?

What you learned: In this modeling task, I learned that typing the formulas of revenues, costs, and profit in Geogebra directly could make graphs. Also, some formulas in US English is different from UK and AU English in tying in Geogebra. In US English, MaxProfit = Extremum [Profit]; in UK and AU English, MaxProfit = TurningPoint [Profit].

Matt W.

Task 5.4: The Aircraft Wing

Keywords
Constants, cross-sectional, symmetric, area

Content Level: Algebra or Calculus

Description
The task asks us to determine values for the constants within the the function in which the designers used to create the wing shape. Then calculate the area of the cross-sectional profile of the wing.

Overall: 3
Engaging: 3 **Difficulty:** 4 **Math Content:** 3

Objectives:
- Create function that represents the profile of the wing of aircraft.
- Calculate area under curve in order to find area of cross-section.

Technology: Geogebra is used to fit a curve and determine the constants that allow for a proper representation of the airplane wing. Fitting a function via parameters is used within this task. Another geogebra tool that is used in the task is the integral function.

Discussion: The task was interesting in that it allowed the person completing the task to explore how the differences in constants can impact a function. The task takes a 4th degree polynomial in order to fit the data with validity. This being said the grade level it is introduced in could vary depending on work with higher degree polynomials. My thoughts on the task is that the nonmathematical portion is not that engaging thus students may not be as inclined to explore.

Anticipated Student Difficulties: The technology use of parameterizing the function to fit the data was slightly difficult to implement in this task.

Ideas about Implementation: The task could be a good exploration for students learning about integration and connecting these ideas to a real life context of integration.

What you learned:
- How to fit a function by introducing different parameters for the data set.
- Use of the integral tool in order to calculate area under the curve.

Abigail Johnson

Task 5.4 Modeling an Airplane Wing

Keywords

Integration, reflection, area, polynomial fit

Description

In this task, students are given a set of data points acquired from placing the cross-section image of a model airplane wing onto a coordinate plane, and a polynomial of degree 4 that has been determined by experts to model an ideal airplane wing. They use this data and the integration function in GeoGebra's CAS to determine the total cross-sectional area of an ideal plane wing that fits the given data points.

Overall: 3

Engaging: 3 **Difficulty:** 2 **Math Content:** 3

Objectives:

- Students will create a model of an airplane wing by generating a fourth-degree polynomial fit to a set of points.
- Students will find the area of the airplane wing by taking an integral and doubling it
- Students will be able to use the Polynomial Fit, Intersect, and Integral functions on GeoGebra.

Discussion: This task, while it has the potential to be interesting, lacks mathematical connections or rigor. Straight-forward and simple in its directions, this task could be easily completed by students of almost any age. The instructions to fit the curve and find the area are carried out without much connection to the formula given in the Task, and the modeling itself is simple and static. Students do not model any changing phenomenon or item that requires interpretation; instead, they perform route calculation to find a set answer to given questions.

Anticipated Student Difficulties: Students may struggle with the fact that the values are so small, and clearly do not represent the actual size of a plane wing. They would likely have similar issues interpreting the final value for the area. These difficulties could be avoided if values were to be different, or if units were to be given for either value.

Extensions: This task could be extended to have students optimize the area of the wing, or create a new wing with the same area that takes a different shape. These tasks would be of a high cognitive demand and force students to utilize higher-level thinking and interpretational skills. The task could also be expanded by having students examine how the area changes as the wing warps to gain or reduce lift during flight.

Alternate Uses Mathematically: Alternatively, this task could be used to teach other methods of finding area, such as Reihman Sums.

Ideas about Implementation: This task should be implemented at the beginning of students' exploration of the concept of Integrals as a means to find area under a curve.

Trevor DeBono

Task 5.6: Volume of a Pear

Keywords
Volume, 3D, roots, functions, integration, displacement, scale

Overall: 3
Engaging: 4 **Difficulty:** 2 **Math Content:** 3

Objectives:
- Students will be able to understand how to find the volume of a 3D object
- Students will see mathematics with real-world applications
- Students will need some understanding of roots of functions and integration

Discussion:
One can find the volume of a pear, or any object, in multiple ways. A suggested way is to determine the amount of liquid displaced when an object is introduced into a container of water. On the other hand, if a student uses an application like Geogebra, an image of the object with correct scale is all that is needed to calculate the volume of that object.

Anticipated Student Difficulties:
Importing the image with correct scale is the most difficult thing.

Ideas about Implementation:
Will work great to use this as a resource of learning following a unit on calculating volume or an experiment similar to the one revealed in the discussion.

Task 5.8 Exercise: Determining Oxygen Uptake from Power

Keywords
Curve, area, integration, functions, free-form

Content Level: Algebra 1: Functions, Pre-Calculus/Calculus: Area under a curve/Integrals

Overall: 4/5 This task could be used among a few different classes. High math content and relatable to students. Not extremely difficult.
Engaging: 5/5 **Difficulty**: 2/5 **Math Content:** 4/5

Objectives:
- Be able to find the area under a curve which will determine oxygen uptake
- Be able to see which strategy will most likely win during a race and prove this mathematically
- Be able to graph data as a series of functions

Discussion: This model was actually very interesting to me. It wasn't too difficult, and it seemed to have a lot of math content to it. There were several different data sets in this, which were graphed as functions. This could be used for an Algebra 1 Class, in showing how different data sets look. In this, you were also to find the oxygen uptake for generalized power, as well as for two specific runners. Once the original oxygen uptake data set was plotted on the graph, you were able to create a function of the oxygen uptake ability as a function of power. You then were able to use this to find the oxygen uptake ability for two runners, each with a different running strategy. The goal was to see which runner would use less oxygen, which would then lead us to believe that is the runner that is in better shape, and will end up winning a race between the two runners. In this, you had to find the integral in order to find the area under the curve of the oxygen uptake function for both of the runners, in order to find the total oxygen uptake. This part can be geared towards a calculus or pre-calculus class. This model also shows you how to free-form a line onto a set of data points in Geogebra, which may be useful.

Anticipated Student Difficulties: The one main difficulty that I had that I anticipate students having is that it can be hard to make sure that you are working in the right graphics area. I found this very frustrating. Another difficulty that students may have is they might have a hard time understanding what the area under the curve means if they are in a lower level math course.

Alternate Uses Mathematically:
This could be beneficial for students who are athletes who are wondering what actually happens when they are competing. It can be related to all students who play sports.
You could use this to introduce integrals to students and to help explain actually what is happening when you calculate the integral, or the area under the curve. It is nice to see on Geogebra, since the area will be shaded.

Ideas about Implementation/Experiment: If you had the ability to, you could have students do the experiment with their own data. You would need a tracking device that would measure student's power output, so it might not be able to be done in a school that doesn't have extensive resources.

Task 5.9: Bicycle Reflectors

Keywords
Movement, circles, distance, circumference, distance

Content Level: Algebra 1 or 2

Description
In many countries, side reflectors must be mounted on your bicycle when you ride in traffic. These reflectors are often mounted on the wheel spokes. When you ride the bicycle, the reflector moves up and down on the wheel. What curve will a reflector follow if it is mounted three quarters of the way out from the hub toward the tire? Students may also look into the question to how far the reflector moves for every mile that the bicycle moves.

Overall: 4

I really liked this problem. The programming aspect was easy to follow, and even when I got tripped up I was able to conceptually rationalize a few changes in order to make my design look like the one the proposed solution suggested. Most students can recall how a bike wheel turns and the restrictions that would exist in real life compared to that of the problem. It leads to good extension questions (can a spoke actually be far out from where the tire is?).

Engaging: 4 **Difficulty:** 3 **Math Content:** 3

Objectives:

- Students will connect the movement of a bicycle wheel to general circles on a graph.
- Students will be able to measure how far a reflector moves for every mile that a bike moves.
- Students can adjust modeling understanding to real-world applications.
- Students will find the impact of where a reflector is mounted by comparing the curves of movement at different positions.
- Students will recognize when time and distance are factors in making sense of a graph.

Technology Requirements: The problem is very dependent on student's ability to utilize GeoGebra, and leaves little room for a different solution method. The requirements within GeoGebra are limited, and could be followed easily. Students can even change some of the parameters without changing the problem itself, or the answers. Depending on the solution method the students follow could lead to problems with the second aspect of the technological requirements. Finally, I feel that the technology requirements allow for students to thoroughly make sense of what is going on in the problem. They are not so challenging that the mathematical aspects are lost.

Anticipated Student Difficulties: I suspect two challenges that students could have while working through this problem. First is that input methods. While the problem is okay enough to follow along, some of the inputs are challenging and students need to be very precise in the way they follow any instructions. I say this because it took me three tries to correctly 'define' the element “C” in order for GeoGebra to replicate what the figures in the book showcased. The other problem I anticipate was briefly mentioned above. There are two results for the first part of the proposed solution, and depending on which one students follow could lead to problems. Since the second part closely resembles one of the solutions from the first part, it is not challenging for the instructor to connect the two. However, if students followed the other solution for the first part, it might lead to confusion and more questions by the class. In order to get around this, I suggest that the educator either work through both paths, or have the students focus on one of them. That way either you have the background in order to answer any questions, or the whole class is on the same page.

Karlie Goretski

Task 5.10: Cardiac Output

Keywords
Concentrations, polynomial, fit, area, integration, change, linear function, logarithmic function, polynomial function

Content Area: Calculus

Overall: 4 (This task engages students to think about calculus and begin to learn how to find the area under a curve. It also allows students to experiment with "cardiac output" and graph the cooling of temperature after being injected into a catheter)
Engaging: 5
Difficulty: 3
Math Content: 4

Objectives:
- Using time and measurements of concentrations to determine the cardiac output
- Plotting points to determine the fit of a polynomial
- Determining the area under the curve
- Determining how the area under the curve changes based on the fit on a function, including linear, l logarithmic, and seven-degree polynomial
- Experimenting with a catheter to help visualize the task and relate it to real-life scenarios

Discussion: In this task, students are asked to analyze data to determine an individual's cardiac output. By plotting points for the time in seconds and the amount of concentration of dye in the blood, students can use GeoGebra to use either a polynomial or a logarithmic function to fit the curve. Then, they can determine what the area is under the curve. Furthering this, we are doing a realistic experiment in class to allow students to find the cardiac output of an individual.

Anticipated Student Difficulties:
- Knowing which model function to use
- Using GeoGebra to determine the area under the curve
- Knowing what cardiac output means
- Using GeoGebra to determine the cardiac output
- Understanding and using logarithmic functions
- Understanding integrals

How Can We Help With Student Difficulties?

- Allow students to experiment with different degree polynomials to determine the best fit line for the curve
- Give a brief introduction to GeoGebra and how to use polynomials and logarithmic functions to model the data
- Give students a brief background on cardiac output
- Have students google cardiac output before introducing the task
- Give students a brief introduction on what integrals are

Technology:

- GeoGebra
- Using a spreadsheet to plot points to find a curve
- Using polynomial and logarithmic functions to model a curve
- Finding the area under the curve

Matt W.

Task 5.11: Medication

Keywords
Area under the curve, function of time

Content Level: Pre-Calc, Calculus, Algebra

Description
The task asks you to calculate the L-DOPA level within a type of medication over a given period of time after the medication has been taken.

Overall: 3.5
Engaging: 3 **Difficulty:** 4 **Math Content:** 3

Objectives:
- Determine a function that fits the data of L-DOPA levels as a function of time.
- Determine the time it takes for the concentration of L-DOPA is below 10 ng/ml.
- Calculate the area under the function curve and interpret the area.

Technology:
- Geogebra fitting a function to a set of data points.
- Using the integrate tool in order to calculate the area under the curve

Discussion: The task is an interesting way to explore a real life usage of integrating a function. It could also be used as an activity to explore intersecting of lines, and interpreting what the intersection means.

Anticipated Student Difficulties:
Determining which function fits the data set the best.
Creating a function by parameterizing the characteristics of the data.
Interpreting the real life meaning of the area under the curve once calculated.

Ideas about Implementation:
Exploring real world applications of integration.
Connecting intersections of lines in context with a real life scenario.

What you learned: The idea of using multiple functions including linear functions in order to fit a set of data and then integrate under each piece.

Task 5.13: Temperature Change

Keywords

Temperature, periodic change, trigonometric functions, Fourier series.

Content Level: Calculus

Description

Students need to represent the temperature and the change in temperature by building a model that fits the data and that describes the change in temperature.

Overall: 3 The task explores interesting connections about ways to improve a model with the use of mathematical concepts, however it doesn't have the students use the model they have made to find new results or make conclusions.
Engaging: 3 **Difficulty:** 4 **Math Content:** 3

Objectives:

- Find model that fits the data.
- Understand ways to make model more precise.
- Understand adding overtone and the periodic relationship of weather.

Technology: GeoGebra is helpful to make precise models of the data quickly. Therefore, the students can make changes to their model to see how the models are improved by the changes they make.

Discussion: The task introduces the topic by connecting it to the fact that increasing the degree of a polynomial function improves the model of the data. The task explores how to do the same thing but when models are using trigonometric functions instead of polynomials. It discusses the idea of adding overtones with different frequencies to improve the model and connects it with Fourier series. It also connects it to the topic of temperature since there are various phenomena that affect the weather periodically.

Anticipated Student Difficulties: Students might see that the model is enhanced by adding these overtones to the models of the data but not understand mathematical why this happens after finishing the task. To avoid this while student are working the teacher should monitor their conversation and bring up their ideas their ideas to discuss as a class. Then the students can research the questions they have from the activity and discuss it for the next day.

Ideas about Implementation: I would start by having the students discuss how temperature changes every year so that they make the connection to periodic motion. Then I would let the students get started on building their model. Later I would have the student talk about periodic phenomena that affect the temperature so that they think of ways to improve their model. After they improved their model I would ask them what they tried and if it work to discuss their ideas and questions.

Extensions: I would have the students think about what aspects can affect the change in temperature and find the data for those years to see how it could be affecting the average temperature recorded. They can see how different periodic phenomena that affect the average temperature are also being affected.

What you learned: Using overtones with trigonometric models.

Chapter 6

Using Differential Equations

Task 6.1: Time Taken For Water To Chill Out

Keywords
Temperature, cooling, flow, predictions, extension, variables, Newton's Law of Cooling

Content Level: Algebra 1

Description
Analyze cooling data to understand how the type of flow with the contact surface can affect the data and model by varying a specific variable. The students will use the model to estimate the when the drink reaches 26 degrees Celsius.

Overall: 3- It takes time to find which model you should use to best fit the data but most of the functions that are used in Geogebra are simply typed in the input bar or using CAS.
Engaging: 3 **Difficulty:** 3 **Math Content**:3

Objectives:
- Have students comprehend another factor that can affect Newton's Law of cooling.
- Understand how varying a variable can affect a model.
- How to find information using a model and understanding how exact that value is.
- Use technology to be able to model a relationship.

Discussion: The task is related to Newton's cooling law. The sketch makes the point that sometimes you must try multiple functions and fits to find the best model. It also includes the idea that extrapolation of data, or drawing conclusions from the model about something that will occur outside of the measured data, is more challenging and can result in a large variation.

Anticipated Student Difficulties: Having students making estimation can be difficult when it's far away from the recorded data so students might get frustrated when they are getting different answers. Remind the student that they need to be able to justify the validity of their model.

Extensions: The first extension could be posing a question to the students about what happens when you start the experiment off with two different cups with different starting temperatures. Ask them which cup do they think will cool faster, and which will cool more slowly. Another extension of this is asking students why pools have to be constantly heated in order to remain at a constant temperature.The last extension is that ask students to think about the reasons behind the temperature could never go to 0.

Ideas about Implementation: Before letting the student start the task have a discussion about how the student think they should model this data so it gives the student some ideas to work with.
This experiment could be implemented in a high school calculus class as a basic introduction to the function of Ordinary Differential Equations, but with the support of the CAS System so that students do not need to solve these equations themselves to understand their usefulness.

Experiment: Have student measure the temperature of hot water cooling down to room temperature, or repeat the experiment but in one cup the student stirs the water and in another they don't.

Technology: Geogebra is useful because it allows students to test various models that will give them the "best" answer to estimate when how long it will take to cool to a specific temperature

What you learned: The CAS system of solving an ODE was a new idea introduced.

Task 6.2: Moose Hunting

Keywords

Population, carrying capacity, Verhulst Model, ordinary differential equation (ODE), fit

Content Level: Algebra 2 or Precalculus

Description

Talks about population control/change in regards to the "carrying capacity" parameter. Bring's up the Verhulst's model that claims $y' = k * y\,(K - y)$. The question ask to solve the equation and fit the solution to the information about the moose. Then, adjust the model to consider the hunting. Finally, find how many animals can be shot each year so the herd size remains stable and is not at risk for extermination.

Overall: 3

We think that the problem could be used very well in the classes mentioned above. While parts of the problem are super challenging, other parts are quite easy to follow and make sense of. We say this because while working through the solution methods we only had trouble with the third process. Students could potentially, easily follow the first two. We also discussed how in areas where hunting is normal, or moose/deer are talked about frequently, students would be very engaged and interested. However in a place, like south Florida, students would be very disengaged. Finally, we have a little disagreement as to how much "math content" was utilized in this problem and decided to take the average. Stephanie thought the mathematics was very apparent due to the use/requirement of looking at the slope fields. Amber thought it did not require as much mathematical content because the students themselves were not having to find the second derivatives (the slopes). Additionally, since the question does not define what all the different variables are in the context of the problem, we are concerned if students will be able to understand their mathematical properties.

Engaging: 2 **Difficulty:** 4 **Math Content:** 3

Objectives:

- Students will be able to solve a differential equation.
- Students can connect Verhulst's model to population and carrying capacity of a given animal
- Students can adjust given models to connect to other parameters (when they are taking into account the animals that are hunted each year).
- Students will be able to compare their results for the given population parameters to other population parameters we might give them.
- Students will gain a better understanding of "carrying capacity."

Technology Requirements: This task has only a solution of using Geogebra but in multiple ways. It used the idea of symbolic, directed, numeric or using this as an exploration for further understand patterns of carrying capacity. This meant the students had to create sliders or use CAS to obtain the answers. If they had a exact amount of moose and a rate they could see this graph on a graphing calculator but not get the exploration that Geogebra offers. .

Anticipated Student Difficulties: Students might have problems understanding at first where to start. The section says to start with solveODE[] command that will probably be new for most of the students. They do provide a link to better understand what this means but I would guess the students would not use this link. Students are able to walk through the solution but they do not do a good job at explaining what they are actually doing. They do not describe what K is until halfway through the problem and k is never actually fully explained. And there appears to be glitches at the end that make the slope field not work well. I would explain to the class what k and K are and what point A is as well. I would also explain what a slope field is and reasons why the slope field would not work. (no slope)

Bowei Zhao
Yaming Liu

Task 6.4: Skydiving

Keywords
maximum, velocity, freefall, differential equations, integral, area, sliders, peregrine

Content Level: Algebra, Calculus, Physics

Description

There are several problems in this modeling task to solve. The first is to find the longest jump possible if we assume the skydiver also must have enough time to release the parachute and lower the speed to secure a safe landing, with time to maneuver to the intended landing area. The second is the distance of skydive last if we assume that the skydiver immediately after the jump dives head first with arms closed to the body, and therefore achieves a terminal velocity of 320km/h. The last problem is to solve the time takes before the peregrine reaches 90% of its terminal velocity.

Overall: 4
Engaging: 4 **Difficulty:** 4 **Math Content:** 5

Objectives:

- Students are able to understand the concepts about the free fall.
- Students are able to solve the differential equations using Wolfram Alpha.
- Students are able to know the integral is the area under the curve.
- Students are able to adjust the sliders of m and k to make the area to be 3200.
- Students are able to learn the peregrine.

Discussion: This problem set up as no matter skydivers jump from 10000m or 5000m, the terminal velocity will be the same. However, I think this might not be correct. As I thought, the velocity of jumping from a 30 meters' height building is different from the velocity of jumping from a 300 meters' height building. I talked to Professor Bae, and he said that this is how the problem set up. However, we did not know if this setup is correct or incorrect.

Anticipated Student Difficulties: It is very hard for students to think of the hyperbolic functions are nice applications of exponential functions. Also, most students will not know the formula of tanh(x). Therefore, students need to learn the knowledge about trigonometric functions in order to complete this task.

Extensions: We can set up the conditions that the terminal velocity is not the same as jumping from different height. In addition, after talking to Professor Bae, Bowei and I decide to extend the task to be a man jumping from a plane, but with a rope tied to the man. This man will release the parachute about the height of 1000m. In this case, the man does not only have the gravity force and air friction, but also the pulling force from the rope. With these conditions, we want to compare the results with the results in this modeling task and see the differences between them.

What you learned: In this modeling task, I learned that Wolfram Alpha could solve a differential equation based on initial conditions. Basically, if you type in the formulas to the wolfram Alpha with initial conditions, the differential equation will show up in the following box.

Karlie Goretski
Sam Barra
Sakthi Shanmugasundaram

Task 6.5: Flu Epidemics

Keywords
SIR model, differential equations, variance

Content Level: Statistics/ Predictions and Differential Equations

Overall: 4 (This task engages students to think about the how human health is affected with a real life scenario. It allows students to see how the numbers change and make predictions with a realistic situation)
Engaging: 4
Difficulty: 3
Math Content: 3

Objectives:

- Using the SIR model to organize differential equations to describe how the number of susceptible individuals, the number infected, and the number of immune/recovered individuals varies over time.
- Being able to create an SIR model for a real life scenario

Discussion: In this task, students are asked to look at the correspondence between number of people who are vaccinated and how a disease will spread. They hypothesize that if enough people are vaccinated then the disease will not spread. They are then asked to make a SIR model to see if they can determine the number of vaccinations that will stop the spread of disease. The overview of the proposed GeoGebra solutions include constructing a graph that includes the parameters of number of deaths per year and weeks.

Anticipated Student Difficulties:

- Students may have difficulties with using a SIR model to model the spread of flu epidemics in Geogebra
- Students may not know what a SIR model is
- Students may not know what differential equations are or how to solve them algebraically
- Students may not know what initial values to use for the differential equations
- Students may have difficulties calculating coordinates for the image's vertices from the coordinate systems position

How can we help Student Difficulties?

- Cover SIR model beforehand
- Have students look up what a SIR model is before task
- Set initial parameters as a class
- Cover differential equations beforehand
- Have students work in groups to work together to solve Geogebra issues

Technology:

- Geogebra
- Mathematical formulas that geogebra can compute to help with sliders and change the numbers on the screen to make different predictions
- Use a spreadsheet to compute the various differential equations

Megan Campbell
Gadea Ortiz
Matt Wisniewski

Task 6.6: USA's Population

Keywords
Population, quadratic, exponential, logistic

Content Level: Algebra 2 and up

Description
The task asks you to make projections for the population growth of the US far in the future and make a model representing the change of the rate of change over time. Make conjectures in terms of the projections that you create.

Overall: 3, Overall this has a rating of 3 because the context is not very engaging for students. Additionally the math content as it connects to the task is not too involved. The task overall is a little challenging because developing the multiple models is after the first two but the comparisons between them are valuable.
Engaging: 3 **Difficulty:** 4 **Math Content:** 3

Objectives:
- Students will be able to organize data in a way that it can be analyzed
- Formulate different functions in terms of single variables that represent the data set
- Create mathematical models that represents the data
- Students will be able to analyze the validity of their models in comparison to other models.
- Students will be able to make predictions about future populations based on their given models.

Technology: Implementing the technology aspect of the task is beneficial because it allows the students to model the data quickly and accurately. The spreadsheet element of using technology allows students to engage with the data set which in turn helps students start to work through the task. Without the use of technology finding the sum of squared errors would take much longer for each model and therefore comparing the models would be much more difficult.

Anticipated Student Difficulties:
- Being able to correctly input data
- Connecting which projections are related and accurate
- Fitting functions to the data modeled

Ideas about Implementation: If this task were to be used in a lower skill class, like in an Algebra 1 course the proposed solutions 1 and 2 would be expected. Once the students get to proposed solutions 3 and 4 derivatives and precalculus is involved thus the difficulty level increases drastically. These can either be omitted entirely or only implemented in a higher class level. Additionally asking students to predict the population in a certain year would be a good task for students in an algebra 1 course but in higher courses I believe this can be distracting for the overall goal and can be omitted.

Extensions: By giving students another country's population set and allow students to compare and contrast the two data sets. This would give students practice making comparisons in mathematics with evidence from their models.

Abigail Johnson
Matt Champagne
Rachel Hoard

Task 6.7 Cheetahs vs Antelope: A Game Of Survival

Keywords

Differential equations, difference equations, cyclic model, numerical solutions, making predictions, validity

Description

In this task, students use different GeoGebra models to determine the trend in populations for a number of cheetahs and antelopes in a given area. Students must determine whether or not either animal is at risk of dying out based on these models' predictions, and evaluate the validity of different models and parameters that may impact this.

Overall: 3
Engaging: 4 **Difficulty**: 5 **Math Content:** 4

Objectives:

- Students will use numerical methods to solve a difference equation, given initial conditions
- Students will create a model of the relationship between cheetah and antelope populations to determine whether either animal will die out
- Students will use GeoGebra to solve a differential equation.

Discussion: This task instructs students to use a set of data and equations that describe predator-prey relationships in order to determine whether or not cheetahs and antelopes will die out. Students utilize a sort of trial-and-error technique in manipulating parameters to get the curve to match measured data, using both a relationship between pure numbers of predators and prey, and a relationship involving the ratio of the two populations. The task concludes with three models, each having their own advantages and pitfalls and without a clear "best" choice, leaving the decision up to the interpretation of the students.

Anticipated Student Difficulties: The task itself states that "to simultaneously adjust four different parameters manually to fit a graph to data can just about drive you insane." The fact that students must slowly and systematically manipulate four parameters in a trial-and-error type method could be frustrating at best and completely derail their work on the model at worst.

Extensions: This task could be extended by asking students to consider how the model would change if other factors affecting population (such as uneven birth and death rates, conservation efforts, poaching, natural disasters, etc.) would impact the appearance of each of the models.

Alternate Uses Mathematically: This task could also be used to teach scientific methods and sources of error.

Ideas about Implementation: This task involves more difficult mathematics and careful and informed manipulation of parameters, which makes it more likely to be utilized in a high school or college calculus class or other upper-level math in which students are being introduced to the idea of two variables interacting in a setting involving the use of differentials. It should be implemented after students have acquired a knowledge of basic calculus concepts but before they begin to get into full-on study of differential equations.

Experiment: There are limited opportunities to experiment with this problem outside of theoretical GeoGebra-based "experiments" or thought experiments about population sizes.

What you learned: In this task we learned just how different models of a data set can be when they take into account different factors, and how data and differing factors can be included or excluded in order to manipulate your findings. It was amazing to me that one model could clearly show that populations would be sustainable long-term and another model could show one or both species dying out very quickly. I think this would teach students to be critical of even the most scientifically and mathematically-sound arguments, as even the most technically correct models can still be manipulated, to a certain extent, to produce drastically different results.

Cayla Pearsall
Joseph Brent Nutt

Task 6.8: Amount of smoke left in a smoking break room

Keywords
Differential equations, parameters

Content Level: Differential Equations, Calculus

Overall: 4/5
Engaging: 4/5
This task was very engaging because the task requires that you pay close attention to what you are typing into geogebra.
Difficulty: 4/5
This task is difficult for a student that had never been introduced to differential equations.
Math Content: 4/5
This problem is very focused on solving and staying engaged in the math.

Objectives:
- Student will be able to determine the amount of a gas left in a room after a certain amount of time.
- Student must be able to use differential equations with various parameters.
- Student could use geogebra or wolframalpha to solve.

Discussion: This task was about figuring the leftover amount of benzo pyrene in a room after people have smoked in it. The students have to create a model to show how the amount of smoke changes over time starting the room off with fresh air. The students have to look at what the room is like after a week straight of people smoking and then after 4 weeks straight.

What you learned: Pay attention to what is being put into each graphics window to make sure it is in the correct one.

Ryan Gronwick
Trevor DeBono

Task 6.9: Alcohol Consumption and Sobriety Over Time

Keywords
Differential equations, rate of change, parameters

Overall: 4.33 (average of three ratings below)
Engaging: 4 stars (the task is something that got my attention and has real world value to it)
Difficulty: 5 stars (works with differential equations which can be difficult)
Math Content: 4 stars (an application of differential equations that requires conceptual and contextual thinking)

Objectives:
- Understand differential equations and rate of change
- Use Geogebra to model alcohol intake vs. time
- Vary parameters such as the intake of food and continuous alcohol intake
- Determine the effects that alcohol has in dealing with intoxication

Discussion: This task uses differential equations to examine the effects of alcohol over time. This takes into account gender, weight, how many drinks are consumed, and how much alcohol is in the standard beverage. The equation considers that the drinks are all consumed in the first minute and it produces a curve to represent the blood alcohol level as time progresses. One component of the equation can be altered to take into account whether food is being consumed as well. An alternate equation can be created to represent the blood alcohol level as time progresses if drinks were to be consumed at a continuous rate. This is a task that might be more relatable to college aged students, however, more and more high school students are drinking nowadays. It is also good to recognize that it is unlikely that high school students will likely not be working with differential equations.

Anticipated Student Difficulties: There are specific commands that the student will need to input that may be hard to conceptualize. This includes defining the derivatives as multivariable functions and solving the differential function using the NSolveODE command. The student will also need to interpret the data after plotting it in the graphics window. This is important because the graph is used to determine how many standard size alcohol beverages a woman can consume initially, if that woman has every intention of driving after six hours. Another difficulty that students may encounter is forming sliders that can be used to vary n (number of standard size drinks consumed) and I (the number of drinks consumed per hour).

Ideas about Implementation: Is the context of this problem worth doing with our students?

What you learned: I think the fact that you can account for the variety of factors is something that I knew was possible, but was not sure how that would be represented. Seeing it first hand and being able to interact with it was cool.

Chapter 7

Geometrical Models

Matt C.
Matt W.

Task 7.1: The Looping Pen

Keywords
Motion, related rates, circles, segments

Content Level: Geometry, proof and logic

Description
Students are given the scenario where a pen is resting on a glass (or something else that will hold the pen the same way) and asked to look at the motion the end of the pen will make when the pen is allowed to slide straight away/toward and through the mug. They are asked to specifically look at if that motion that is made by the end of the pen is part of a circle.

Overall: 3
While this was a very interesting activity for students to work on, I feel that the knowledge gained is diminished by how long it takes to look at this single instance of motion.
Engaging: 3 **Difficulty:** 3 **Math content:** 2

Objectives:
- Students can work with segments and circles to make observations about a situation
- students can make make conjectures about the movement of an object
- students can prove whether or not a point defined by a particular motion will be part of a unique circle

Discussion: This task walks student through the steps of constructing a representation of a pen resting on a mug in geogebra. This allows students to take a deeper look at the motion of the end of the pen and see what shape is traced out when we move the pen to all of its possible locations. Students will see through this task that while the motion of the end of the pen, although appearing circular at first, actually belongs to a different shape.

Technology: The most useful technology available to students would be GeoGebra. The task gives the opportunity for students to look at the motion that is caused by this relationship, and also shows the students some new tools that they can use, such as creating segments and having objects operate on others while not being shown in any graphics.

Anticipated student difficulty: As with most of these tasks, the biggest difficulty I can forsee is GeoGebra itself. At times it can be frustrating trying to get objects to interact with each other the way you want them to and can take some troubleshooting to figure out. That being said, this task in particular is not terribly difficult as there are not that many parts interacting.

Karlie Goretski
Megan Campbell

Task 7.2: Comparing Areas

Keywords
Polygons, areas, triangles, intersection, proof, central area

Content Level: Geometry, Proof and logic

Description
Students are given a picture and are asked to find a relationship and if one is found they need to try to devise a proof for that relationship

Overall: 3 - This task can be very difficult for students. It will require a lot of persistence to find the relationship at hand with the help of GeoGebra. Furthermore, this level of proof will be difficult for someone to create without much experience with proofs.

Engaging: 2 **Difficulty:** 5 **Math Content:** 3

Objectives:

- Students can work with polygons and compare their areas.
- Students can find when the triangles they are comparing have the same area.
- Students can decide what happens to the areas of triangles when a point coincides with them.
- Students can establish a proof for the general case of showing that the areas of three polygons are equal to the area of the central area.

Discussion: This task walks students through comparing the areas of polygons to the central angle and allows students to prove the general case for this. Also, this task allows students to prove special cases, such as when two points coincide with each other, as well as when two triangles that you are not using have the same area. Students will see that changing the perspective can make the general proof easier to solve. This task would be good to help students relate the areas of polygons to each other and establish a proof.

Technology: GeoGebra can be used to investigate the relationship between the shaded areas in the image. Students can practice their GeoGebra skills by creating the image that they are trying to analyze in addition to using it to investigate the problem. Furthermore, the technology can be used to measure the areas of the triangles with the program in order to see a relationship between them. The difficulty with this is a proof of a relationship cannot be made simply by looking at examples of it working. You can only disprove it by a counterexample; however, you cannot solely prove by examples. Thus the logic and proof is to be done mostly without the use of technology.

Anticipated Student Difficulties: It may take students a while to experiment with this task on their own. They may have difficulties determining the proofs of special cases, such as when point F coincides with point A or point C. To combat this it is important to have scaffolded how students should investigate problems they don't immediately know the answers too. They may struggle with determining how the proof of the general case of how the areas of polygons all relate to each other. This will require a lot of experience with logical statements and the structure of logical proofs. Also, if students are not familiar and comfortable working with GeoGebra, it is likely that they will have difficulties with this, especially if they do not read the steps carefully. Building the task itself in GeoGebra would help students to understand how the diagram is set up and would likely improve their GeoGebra skills.

Task 7.3: Explore relationship between the heights, medians, circumscribed, and inscribed circles of the four triangles constructed with four intersecting lines

Keywords

Exploration, GeoGebra Tools, Triangle, Medians, Heights, Circumscribed circle, and Inscribed circles

Content Level: Geometry

Overall: 4, the task is engaging because the students are active participants in the learning process. Mathematically, it introduces different relationships that various properties of triangles have.

Engaging: 4 **Difficulty:** 3 **Math Content:** 4

Objectives:

- Make and test conjectures from the information found
- Understand different properties of aspects of triangles and how they are connected.
- Utilizes the tools of the technology to study various relationships

Discussion: One of the main focuses to this task was that you could make and save a tool that can be reused, so that students don't get stuck on making the circles and can focus on the task of exploring the relationship. Students will have the opportunity to make assertions and test various scenarios using the technology. This task would be a good introductory task to using GeoGebra (or another similar software) to a class who is just starting to use it.

Anticipated Student Difficulties: The biggest difficulty would be creating the circles for all of the different triangles; this difficulty is resolved by showing the students how to build a tool before they begin their exploration. Another difficulty is that students might not understand the relationships that they are supposed to be seeing and exploring unless directed.

Extensions: With the technology the students can view the different examples and compare and contrast the various scenarios. In a group the students can try to explain why some of these conjecture are true. After the students should be able to apply these statements to a logical argument and be able to relate this information with other relationships of triangles.

Ideas about Implementation: The task doesn't have much structure and asks the students to explore many aspects. I would suggest forming groups and have each group investigate one aspect (heights, medians, circumscribed, and inscribed circles) and then have them share what they learned and why it's important with the rest of the class as they test their conjectures as well.

Technology: The technology is very important for this task because GeoGebra allows student to examine different situations quickly to test the conjectures that the students made. Before this task, students should have experience using geometric tools in GeoGebra but shouldn't have too much trouble using the technology.

What you learned: We learned how to make a tool on GeoGebra.

Ryan Gronwick
Cayla Pearsall

Task 7.4: The points on a triangle and how they are related to each other (an interactive figure) including the Euler line.

Keywords
Centers, triangle, Euler line, incenter, centroid, circumcenter, orthocenter, nine-point center, symmedian point, gergonne point, nagel point, fermat point

Content Level: Geometry

Overall: 4.33 stars (average of the three below)
Engaging: 4 stars (The task was fun because being able to create the triangle was fulfilling, and using Geogebra would be very engaging for this task)
Difficulty: 4 stars (It was more so a follow the steps thing. Without the step-by-step, it would have been hard. Also without the Geogebra software the task would be hard)
Math Content: 5 stars (The content in this activity was rich as it provides a large variety of geometrical ideas)

Objectives:
- Students will see the different centers related to a triangle.
- Students will see how the centers of a triangle change as the side lengths of a triangle change.
- Students will be able to see the centers of a triangle related to the Euler line.
- An understanding of GeoGebra is needed. Knowing how to construct figures and applying built in functions that use sliders is necessary.

Discussion: This task relates to the Euler line and it associated points. In the task, you construct a triangle that will show just a few of the large variety of points in a triangle. The points shown in the sketch include the incenter, centroid, circumcenter, orthocenter, nine-point center, symmedian point, gergonne point, nagel point and the fermat point. As slider was created to change the point that was visible through a function already in GeoGebra. This sketch would be really helpful in allowing students to explore the different point on a triangle and see how they relate to each other. A lot of the different points and lines are those we would have seen in MTH 330.

Anticipated Student Difficulties: If students don't know how to work with GeoGebra, this task is nearly impossible. So provided some guided steps if you want them to construct it would be needed. Once constructed, there might be some trouble understanding some of the relationships. There is also a lot of vocabulary that is needed to know what is happening.

Items that stand out: The trace function and the various steps that go along with it seemed very tedious if you have no knowledge on how to change the colors of the various trace aspects, or no knowledge on how to ge trace to animate the entire screen that is displayed.

Alternate Uses Mathematically: This Geogebra file could be saved when done and the students could use it again later on to find more points on the Euler line.

Ideas about Implementation: I think this could be used to help students discover different points. They will eventually need to be told the definitions, but having the students discover things like the fact that the altitudes of each vertex intersect each other.

What you learned: If you are unable to get the trace part of the proposed solution to work make sure that you are hitting enter/return after each entry in the red and blue boxes.

Sakthi Shanmugasundaram
Brent Nutt

Task 7.5: Trisected Area

Keywords
Triangles, geometry, area

Content Level: Geometry

Overall: 3 (Just working with splitting triangles, not sure what to do on after that. Might need to come up with an extension of your own if you are using this task)
Engaging: 4 (will have students talking and working to find many different ways to partition the triangle)
Difficulty: 2 (not that hard to be able to come up with different ideas)
Math Content: 3 (working with different concepts in geometry to help produce solutions)

Objectives:
- How to be able to split a triangle up into equal area
- Seeing multiple different ways to cut a triangle into 3 pieces
- This task relies pretty heavily on GeoGebra

Discussion: There were 2 proposed solutions for this modeling task:
1. Proposed Solution 1
   a. Working with GeoGebra to help you split the triangles and move according to equal area
   b. Allows to change the shapes of the equal areas and come up with different possibilities
2. Proposed Solution 2
   a. Still working with GeoGebra
   b. Another way to partition the triangles that uses the points on the inside and different geometry concepts to recognize how the areas will be the same

Anticipated Student Difficulties: Some students may come up with unique ideas with how to split up the triangle, but they might have a hard time figuring out how to represent it in GeoGebra.

How can we help Student Difficulties? Try to anticipate many of these unique student solutions, so that we can be prepared to help students as much as we can with certain techniques in GeoGebra.

Sam Barra
Bre Rosol

Task 7.6: Spirograph

Keywords
Spirograph, radius, gears, multiple representations, proof

Description
Create a spirograph using geogebra to determine the relationship between the radius of the gear and the amount of petals the finals pictures has.

Overall: 4
Engaging: 4
Spirographs are a fun craft that students could use first as just a creative outlet and then making it using a computer program was very cool to see. It is also something that is relatable after using a spirograph on paper.
Difficulty: 4
The use of geogebra to create the spirograph was challenging but not too hard where students would give up. One the graph is created it would not be hard to create a chart of the radius and amount of petals to see the relation.
Math Content: 3
After creating the spirograph students are required to analyze data to look for patterns and relationships. They are asked to hypothesize and prove their thoughts. This activity requires math without being very obvious.

Objectives:
- Relating a picture to a table
- See patterns on a table
- Create relationships
- Discuss circumscribed and inscribed circles
- Critical thinking of data
- Hypothesize mathematical statements
- Prove thoughts
- Create mathematical questions
- Geogebra

Extension: This lesson could be used in the middle of a sine and cosine lesson. If students have already seen what a sine and cosine graph look like the spirograph can be related to these graphs. By combining these graphs some spirograph designs are created. This lesson could lead into a deeper discussion of how and why this happens so students can better understand these functions and their relationship.
A further discussion can be had about frequency, amplitude and diameter. Students can discuss how they think these things affect the final picture and why they have those changes on the spirograph.

Task 7.7: Studying Geometric Oscillatory Drawings with the Harmonograph

Keywords
Harmonograph, circles, lines, intersections, locus, oscillatory motion

Description
Recreate this homemade harmonograph in Geogebra

Technological Objectives:

- Familiarizing students with various Geogebra tools, including:
  Point ; Circle tools , ; line tools , ; intersect ; locus
- Explore assigning animation speed to sliders via object properties

Conceptual Objectives:

- Exploring concept of locus through visualization of oscillatory motion with one arm fixed.
- Explore geometric properties of drawings.Test conjectures on factors which affect types of pictures drawn (for example, shapes of pictures, presence and numbers of nodes)
- (For more advanced grade-levels) Exploring parametric models of oscillatory motion (for example, Lissajous curves, and dampen Lissajous curves) (http://mathforum.org/mathimages/index.php/Lissajous_Curve)

Anticipated Student Difficulties:

How to create locus and why does it look like that

> Provide the definition of locus, then note the locus tool takes in argument of two points. Now try using the locus tool on any various two points in the construction. Then, manually move one of the two points and note what happens to the other point. What remains invariant as you manually move the point?

Trouble assigning the speed to the second disk

> Ask what happens if the disks are spinning at the same speed? (Only a single closed curve is drawn!) Thus more interesting pictures are drawn when the speeds are different. It would be easier to track the difference in speeds with a ratio. Thus the second slider represents the ratio of two speeds. If the name of the first slider is “Speed” and the name of the second slider is “ratio”, then the speed of the second disk ratio times speed, typed as “Speed ratio” in object properties.

Some Extending Questions:

- What functions can we generate from this harmonograph? (For example, what happens if we plot various distances between various points against increments of time?)
- What happens if the disks aren’t circular, but oval-shaped? Square-shaped? Other shaped???
- What happens if the entire disk(s) themselves are moving (linearly, nonlinearly) as the

picture is drawn?
- What happens if we vary the length of the arms?

Amber Smith
Trevor DeBono

Task 7.8: Folding Paper

Keywords
Right triangle, triangles, maximum, folding, area

Description
An A4-size paper is folded so that one corner lies on the long edge of the paper. A small right-angled triangle is formed in the corner as seen in Figure 7.45. How can you maximize the area of this triangle?

Overall: 3
This problem allows for the possibility that all students be successful with little guidance from the teacher. The solution can be determined from GeoGebra, or by hand, so the problem can be presented in any classroom.
Engaging: 3
The solution methods are easy to understand and some students might fins the idea of being able to "fold" paper on a program as exciting.
Difficulty: 3
The challenging aspect of this problem deals with students knowing when to do certain things: reflecting over line/how to do so, connecting a point so that it changes three different triangles at the same time, etc.
Math Content: 3
Working with different areas and figuring out the maximum, in either solution method.

Technology Requirements: Computers equipped with GeoGebra, students with the capability to work with and manipulate lines and points in GeoGebra.

Objectives:
- Students will be able find the area of a triangle created by folding paper.
- Students will be able to find the point at which folding paper gives a maximum area.

Anticipated Student Difficulties: Not necessarily a difficulty, but if students scale the "paper" into
GeoGebra differently from one another (say one group uses 21x29.7 cm, while another group uses 8x11 in) but they do not specify their units, the end results will be different. Teachers should note the possibility of different results in case this issue arises.
An actual issue would be the level at which students are comfortable with the technology. If students do not understand how to implement the different steps within GeoGebra, they might get lost or need the instructor's "helicopter help." To explain, a student might be so lost on how to set up the different triangles so that the shapes resemble a piece of paper (where each shape

changes a bit when
one is changed), the student might need the instructor to give step by step
instructions, or even help them set it up.
In the first solution method, students might find difficulty in figuring out the “exact” solution. Students might find a couple solutions and then just guess what the final answer is. This would not help them in understanding how to find the maximum area correctly, and could require additional assistance.

Abigail Johnson
Yaming Liu

Task 7.9: The Locomotive

Keywords
Circles, trigonometry, sine, cosine, rotation, segments, periodic motion

Overall: 4
Engaging: 5 **Difficulty**: 3 **Math Content:** 4

Objectives:

- Students will determine whether the motion of a locomotive wheel and piston follows the pattern of a cosine wave or a sine wave.
- Students will use GeoGebra to create a simplified general model of a train wheel and piston

Discussion: This task seeks to have students create a general model of a train wheel and piston in GeoGebra, and then use that model to determine if the motion of the piston follows the pattern of a cosine or sine curve, and where the piston has the greatest velocity. This is a tricky construction in GeoGebra that's accomplish-able but requires diligence in following the directions in the book very clearly. The model is easy to see and relate to the graph, as the construction allows students to observe the curve being formed as the train wheel rotates.

Anticipated Student Difficulties: One student difficulty that could arise is the precision with which directions had to be followed to get the construction to work, which we encountered problems with.
The other thing that may be difficult to students would be the fact that GeoGebra's construction is oriented vertically and a real train wheel and piston would be oriented horizontally. This might conflict with students' existing schemas of trains and would interrupt the process of drawing connections.

Extensions: This task could be extended by having students make predictions about what changing the size of the wheel, its speed, the piston's original position on the fixed track, or another parameter would do to the graph, and then using GeoGebra to test their hypotheses.

Ideas about Implementation: This would be a good task to implement in a precalculus classroom as an introduction to periodic motion, especially if exploratory tools like motion detectors for experiments are unavailable.

Experiment: This task could easily be adapted to become an experiment, using a wheel of any type and a rigid, straight object such as a ruler or a piece of pipe. Students could attach the pipe to the wheel and to a fixed track, and record its horizontal position at different points by hand using a ruler and a protractor.

What you learned: I learned how to use the "Animate" tool in GeoGebra to make points move continuously in order to see patterns

Yaming Liu
Abigail Johnson

Task 7.9: The Locomotive

Content Level: Algebra

Description

Create a model and analyze whether the piston's horizontal component generate a sine or a cosine wave, and whether the piston has its highest velocity at the center of the cylinder if the cylinder is horizontal.

Overall: 3
This modeling task does not require much math content, but more skills with using Geogebra. It is very important for students to know how to make this model. If students can make the model successfully, they will not have any problem on analyzing.
Engaging: 4 **Difficulty:** 3 **Math Content:** 3

Objectives:

- Students are able to distinguish the sine wave and cosine wave.
- Students are able to create a locomotive model.
- Students are able to read the derivative graph and gather the information.
- Students are able to find the roots of the graph.

Discussion: This modeling task is very engaging for students to solve because it talks about the locomotive, which seldom shows up in the math activity. I believe this is a good aspect to introduce the cosine wave and sine wave. I want to know the other interesting real-life situations could introduce this topic.

Anticipated Student Difficulties: It biggest difficulty for students is to figure out the function g(x), which is . In addition, it is hard for students to create this model and make last parts in the Graphics Area 2. Therefore, I think the text should give a multiple steps guide for students to learn to make the model in extra.

Extensions: In the figure 7.15, we can see the piston's horizontal component generate a cosine wave. As teachers, we should ask students the reasons behind the cosine wave, and why it is not a sine wave. What is more, in the figure 7.52, we can see the differences between a pure cosine function and the one we made. Therefore, we can ask students the factors cause these differences.

What you learned: In this modeling task, I learned that after creating the model, Animation on button can make the "train" move. Then, the track of moving (Track on button) could generate a graph to show the pattern. In addition, I learned to use the Point tool and the Intersection tool to make the graph in the Geogebra.

Chapter 8

Discrete Models

Bowei Zhao
Ryan Gronwick

Task 8.3: Determining the endangeredness of the red squirrel

Keywords
Markov chains, transitional matrices, recursive relationship, probability

Overall: 3.6666667 stars (average of the three ratings below)
Engaging: 2 stars (The topic isn't one that I think students would engage with.)
Difficulty: 4 stars (The solution is short and the solution steps are straightforward. Without the solution provided, the task would be more difficult.)
Math Content: 5 stars (The math in this task is valuable to statistics and has good underlying concepts.)

Objectives:
- Students will explore the concept of Markov Chains and of transitional matrices for recursively predicting future outcomes.
- Students will apply data that was collected and find and interpret probabilities.
- Students will need to interpret data collected know how to apply the probabilities found in a way that reaches the task goal. Understanding how to work with and apply matrices is an important mathematical practice here.
- Students will need some kind of computing software as a matrix is raised to a large power. This could be GeoGebra, MATLab, excel, etc.

Discussion: The goal of this task is to predict the population of the red squirrel and to determine whether it is at risk of being endangered. This task incorporates the use of statistical analysis through recursive relationships. Providing a table of varying squirrel populations, the use of probabilities can be helpful in finding future squirrel populations. This is a type of problem you might find in statistics as it requires an understanding of probability. It also requires a certain degree of knowledge in linear algebra as it requires the application of matrices. It is a clear, but challenging way of connecting statistics and linear algebra in an intellectual way.

Anticipated Student Difficulties:
Identifying initial state: what is the initial state in this problem? We could ask students to come up with possible initial states, then work out the problem using those initial states. If we do this, we would see that, in fact, the resulting matrix after 200 years will be the same, regardless of initial conditions! (That is, the initial state does not matter in this problem). To see this, note what happens when we multiply two matrices with: matrix 1 - the probabilities of each row add up to one, and matrix 2 - entries within each column of a matrix are the same.

Recursive thinking in relation to raising power: i.e. why is the probability after n years related to the transitional matrix raised to the nth power? To address this, first make sure raising matrix to a power is clearly defined as repeated matrix multiplication. Now consider an inductive approach. What is the probability matrix after 2 years? After 3 years? Explore how the state of the nth year depends on the state of the (n-1) year. Bring to light the inherent assumption in raising matrix to a power - that the transitional matrix for each successive year is assumed to be the same.

Megan Campbell
Stephanie Young

Task 8.4: Chlorine

Keywords

geometric series, functions, fit, convergence

Content Level: Algebra (functions), Calculus (geometric series)

Description

On day one 15L of chlorine is poured in a swimming pool. After 24 hours 15% of the chlorine content disappeared but another 1L is added to the pool. How much chlorine will there be in the pool after one day, after you added the extra daily liter? How much chlorine will there be after two days? After three days? Is there a convergence?

Overall: 3
This problem was not inherently difficult to understand given the problem posed or mathematically. Only thing that could be difficult is a high level of mathematics must be known to understand the answer that is found by hand using geometric series.
Engaging: 4 **Difficulty:** 3 **Math Content:** 4

Objectives:

- Students will be able to take a given problem and find the data set to go along with given parameters
- Students will be able to practice fitting a model to a set of given data.
- Students will be able to find the convergence of a geometric series.
- Students will be able to interpret the convergence of a given problem.
- Students will be able to manipulate the factors that affect convergence for a goal.

Technology Requirements: This task requires GeoGebra or another dynamic software to model the chlorine problem as posed. A spreadsheet is required to take the given problem and find the convergence overtime. If students are to do this by hand it would take a considerable amount of time, but the spreadsheet makes it a lot easier. They can also practice making the model visually appealing to those who aren't working with the problem themselves. To find the convergence mathematically though with the same problem no technology is required.

Extensions: I think a reasonable extension to this problem would be for students to have a goal amount of chlorine in the pool after say 20 days. Then students will have to manipulate the starting data and the amount added daily in order to get the desired amount of chlorine on day 20. It is possible that the chlorine used could be manipulated since some chlorine may last longer (or shorter) periods of time. All of these factors can be adjusted feasibly using the technology and using sliders in order to arrive at the desired chlorine convergence.

Anticipated Student Difficulties: I anticipate that students are going to struggle setting up their geometric series given a story problem of information. The key will be to think about the amount of chlorine that is started on day 1. Then if students think about the chlorine that is left after each day rather than the amount that is removed they can easily write their convergence. By having students by hand working with their given information and find the first couple days (say 5 days) of chlorine will give students an idea of how the chlorine is changing. Without this firsthand experience working with the story problem can be difficult.

Task 8.5: The Deer Farm

Keywords
Model, Population, Recursive Rule, Stable Population.

Content Level: 7th & 8th Grade

Description
The task asks the students to build a mathematical model based on the populations of deer by analyzing various parameters such as deer born, deer shot, deer dead, year to start hunting, and available resources. The students need to find when to start hunting and how many deer to hunt in order to have a stable herd.

Overall: 3, the students can be active learners in this task, depending on the students the topic of the task might need to be changed. The task has students explore different mathematical aspects of a model.

Engaging: 3 **Difficulty:** 3 **Math Content:** 4

Objectives:
- Understand Recursive Rules
- Build a mathematical model
- Be able to find information and make educated conclusion with the information from the model.
- Notice how slightly changing the parameters affects the results.

Technology: Technology is important because it allows the student to generate the data quickly from the recursive rule using the spreadsheet. It is also useful because the program allows the students to quickly plot the data accurately and see how changing the parameters affects the model. Solving this task would be difficult without using GeoGebra.

Discussion: This task allows student to see how slightly changing certain parameters of a model can have a great affect on the outcome. It also shows that adding the parameter of how resources affect the population can change the model of population size and an anticipated outcome.

Anticipated Student Difficulties: A possible student difficulty is not being able to enter information correctly into the spreadsheet: Students should have experience with the spreadsheet before doing the task, if not the teacher should provide steps for the students to do.

Ideas about Implementation: This task could be used as a real world example to help students understand the implications of changing a parameter. Similar to how A,B,C affect the function Ax^2+Bx+C. It could also be used as a task to help students develop critical thinking skills such as coming up with factors that may impact the calculations.

Extensions: After the students do the task the student can be put into groups and think of a question that can be answered using the current model or extending it. After the student will be assigned another group's questions and will have to explore it and find the answer.

What you learned: One thing that we learned from this task is how to use GeoGebra in a way that we can create/add parameters that affect a function model. Also the idea of representing a recursive model in GeoGebra was new.

Sakthi Shanmugasundaram
Yaming Liu

Task 8.6: Analyzing a Number Sequence

Keywords
Sequences, series

Content Level: Precalculus (series and sequences)

Description
What is the next number in the sequence 8,8,8,32,128,368…? Is there more than one possible answer?

Overall: 4 (I think it helps students to see how to work with a set of numbers in multiple different ways and it gives multiply ways to work with the set of numbers which can be much related to real life scenarios)
Engaging: 4
Difficulty: 2
Math Content: 3

Objectives:
- What is the next number in sequence
- Is there more than one possible answer
- To be able to build a mathematical model
- It engages multiple approaches to solve the next number in a sequence, one using Algebra, one using spreadsheet, one using wolfram, one using the online calculating sequence tool.
- Students need to have creative thinking and try to get as much as possible ways to solve one problem. They might find out that they will get different answers from different ways. So, this also requires students to

Discussion:
Which strategy do you prefer to use in the classroom when you are teaching this topic? Is there a strategy that you will not introduce to students if students do not find it by themselves? For example, the online tool and the wolfram Alpha. Do you think introduce those to students really help students to work on the problems or will you just introduce those to students in some specific situations?

Anticipated Student Difficulties:
For the first solution, students can easily put the numbers in the Algebra. However, it is hard for them to think of choosing a fifth degree of polynomials and then calculate the f(7).
For the second solution, which is putting the numbers in a spreadsheet and using the spreadsheet to calculate the next number of the sequence. It is not difficult for them to put the numbers in the spreadsheet. The one essential part needed to pay attention on is they have to put “=” sign in

front of the expression or the spreadsheet will not work. Also, even though it is easy to work on, it is hard for them to understand why they are calculating the difference unless they already learned the sequence formula to calculate the next term. What is more, we can see that it takes several steps to get the same difference, and students need to generate the polynomials by looking at the spreadsheet. So, this strategy is not as convenient as the first one.
For the third strategy, the wolfram gives a rational function, a quotient of a third-and a second-degree polynomial. The terms are different from the ones we get in the Algebra.
For the last strategy, the online tool is not working when the sequence is “famous”. In my opinion, this is only a checking tool for students to use rather than a problem solving tool.

How can we help Student Difficulties?
There is a mistake in the reading. in page 374, it says “In Cell D4 type =C4-D3”, which is not correct. The correct one should be type D4=C4-C3. This mistake will mislead students. Teachers should do the steps follow the textbook before given to the students.
To explain why the differences calculated in the spreadsheet can get a polynomial, we can use the similar differentiable topic.

What did you learn: I learned there are multiple solutions to a sequence, which I never know before. This is just like there are multiple ways to think about one problem so that different strategies could lead you to different answers. Also, there is not a perfect “right” or “wrong” answer. Along with I learned how to use the spreadsheet to find the polynomial and the next term for a sequence. Even though spreadsheet is more complicated to use, it is still a very good tool, which could do a lot more than I thought before.

Task 8.7: Inner Areas in a Square

Keywords
Square, inner square, ratio, area, inner polygons

Content Level: Algebra 1, Algebra 2, Geometry

Description
The problem gives a few different scenarios in which you can try to determine if there is an inner square created by line segments and asks you to find when those inner squares exist, while also figuring out the area of the inner squares. It asks you to compare ratios between the two squares, and if there is a pattern when you change the subsections.

Overall: 1
This problem would be fairly difficult to carry out in a course. There are many places for error in trying to find the solution, and without a heavy background in GeoGebra students will get lost in the solution. Even going through the proposed solution method I got lost, confused, the programming did not work, and then I did not feel like I was even answering any of the posed questions. This could lead to students giving up on the problem as well as being extremely disengaged because they are so confused. Additionally, there are so many pieces to this problem, I feel like it would take an entire class period just to answer everything and the final part just says "Investigate!"
Engaging: 1 **Difficulty:** 5 **Math Content:** 2

Objectives:
- Students will be able to determine if an inner square exist given specific ways to split a square with side length s.
- Students will determine the ratio of areas between existing inner squares and original squares.
- Students will be able to explain the conditions in which an inner square will not form.
- Students can find different shapes with a square by changing their model.
- Students will be able to compare and contrast the ratios between inner squares and inner polygons of their model.
- Students will be able to explain how the area of the inner square changes based on a change made to the original square.

Technology Requirements: This task heavily relies on GeoGebra, and students' ability to manipulate different programing pieces. The problem has a proposed solution that leads to more questions than answers within the technology use, and does not feel like the question is answered. There are multiple points in which errors can be made, and no input on how to combat the technological errors. When creating a new tool, GeoGebra does different then the book suggest and then you cannot continue with the process. This leads to problems in the students' understanding, and could make them give up the task completely. The technological demand leads to less focus on the mathematical properties of the task.

Anticipated Student Difficulties: I believe students will have a difficult time keeping a clear focus for the entire time. With all the different expectations of the task, it is difficult to remember what questions are being asked. The step of creating a new tool could also confuse students even further. Overall, I feel as if students will not understand why they are doing what they are doing in GeoGebra as this is a very complex task. One way to try and remedy these problems is to not follow the GeoGebra solution method. If presenting this to your class, try creating a template where there is a box with lines from the vertex to the opposite sides. This template should also have the points on the opposites hooked on. From here students could play around with the size of the inner square on their own and not get tripped up on the programming issues. Additionally, instead of having the program compute all the ratios have the students do it by hand. This focuses back on the actual mathematics of the model. Another option would be to have a template where there is just the outter box with a bunch of different, pre-made, lines. Students could drag and place these over the original square and make a bunch of different inner squares. This would allow for more exploration, and could lead to answering more of the questions posed in the problem.

Abigail Johnson
Cayla Pearsall

Task 8.8: Ratio of Inner and Outer Areas in a Triangle

Keywords
Proportion, ratio, discrete, discreteness, triangles, construction, area, hexagons, geometry

Description
In this task, students construct a triangle with three segments connecting its sides and angles to form an inner triangle, and later with six segments to form an inner hexagon. They then compare the ratio of the area of the inner shape to the area of the large outer triangle initially and after changing the positioning of the inner line segments.

Overall: 3.66/5
Engaging: 4/5 **Difficulty:** 3/5 **Math Content:** 4/5

Objectives
- Students will be able to divide a line segment (side of a triangle) into an odd number of congruent parts using GeoGebra
- Students will be able to use GeoGebra to construct a polygon, construct line segments that lie within the original polygon, and measure the areas of both polygons
- Students will draw conclusions about the relationship between the area of a triangle and the areas of several shapes constructed inside the triangle using its parameters.

Discussion: This task has students working with relatively simple but mathematically loaded constructions with lots of opportunity for in-depth thinking. Students utilize an equation provided by the book in order to divide the side of a triangle into equal parts and place points on the segment at specific places. These equations provide a great opportunity to discuss discreteness and why the equation is able to divide a segment into n equal parts, how this equation relates to the distance formula, etc. It also provides students with an opportunity to draw conclusions about similar triangles and constant proportions between shapes with different numbers of sides. This would be a task with few logistical or technology hang-ups, and that has the potential to be very mathematically engaging to students.

Anticipated Student Difficulties: Students may experience difficulty with creating the equation to find the appropriate points on the graph, as these had to be generated based on similar equations utilized in Sketch 8.7.

Extensions: This task could be extended by asking students to examine why we split the side lengths into even numbered portions. The task could be extended by having students select three points on the side length from which to create segments, and examining whether the proportional relationships are still constant between the original triangle and the resulting shapes. The task could be extended by having students create segments that trisect the angles and connect to the opposite sides instead of using specific points along the side segment, and examining whether or how this impacts the proportional relationship between the areas of the triangle and the inner shape.

Alternate Uses Mathematically: Alternately, this task could be used to discuss triangle similarity.

Ideas about Implementation: This task could be implemented relatively straight-forwardly in a classroom as students are studying proportional relationships, nonlinear equations, or as an introduction to the idea of discrete mathematics. This task should be implemented in tandem with Task 8.7, which utilizes equations that students must generalize/build off of in order to create equations used in 8.8.

Experiment: Students could experiment with this concept by constructing their own triangles out of string or on paper. The experiment would involve lots of measurement with a ruler and protractor, and students taking data on the varying areas of the inner and outer triangles as they changed the side lengths.

What you learned: I was surprised at the fact that the areas had a constant proportion for both shapes. I can't help but wonder if the ratio of the area of the inner triangle is proportionate to the area of the inner hexagon and whether it would be proportionate to any other inner shapes constructed using additional line segments.

Trevor DeBono
Matt Champagne

Task 8.9: A Climate Model Based on Albedo

Keywords
Discrete, maximum, minimum, temperature

Overall: 4
Engaging: 3 **Difficulty:** 5 **Math Content:** 4

Objectives:
- Students will be able to use Geogebra to model the albedo of Earth as the average temperature varies
- Students will need to be able to interpret data by multiple representations
- Students will need some background knowledge on climate and weather.

Discussion: This task models the albedo of Earth as the average temperature varies. Albedo is the measure of the amount of clouds that exist in the atmosphere at a given time. The discrete nature of this is that there is a maximum albedo of 0.80, which occurs when the average temperature decreases considerably and results in an ice age, and a minimum albedo of 0.15, which occurs when the average temperature increases and ultimately leads to increase in ultraviolet radiation. By taking into account the solar constant, in addition to the current and future average temperatures, students can model this phenomenon of weather.

Anticipated Student Difficulties: The ability to use the Geogebra application was the most difficult aspect of this task. I believe there are some typos within the instructions which led to this conclusion. Nothing we did could model the exact same result.

Ideas about Implementation: This would be of better use in a science classroom than a mathematics classroom. It would work best within a unit of climate and the atmosphere.

What I learned: The clouds serve an important purpose and depends on the temperature. The albedo is the percentage of radiation that is reflected back to the atmosphere; so the higher the albedo, the less UV radiation that makes its way to the Earth's surface.

Task 8.10: Traffic Jam

Keywords
Statistics, trigonometry, waves, circle, density

Content Level: Statistics, Trigonometry

Description
Finite reaction times lead to domino effects, resulting in density waves, congestions that move very slowly or even not at all. Create a model with the capacity to visualize this type of density waves

Overall: 4
Engaging: 5
This task asks the student to model how cars might move along a road, and see how random variations in speed and spacing can cause a traffic jam. This problem is easily relatable for juniors or seniors in high school because they are beginner drivers. I think they will be interested in seeing how and why traffic jams occur and will look more in depth in the problem.
Difficulty: 4
The proposed solution uses an equation to model the speeds of cars driving around in a circle, but there are lots of steps involved to set up the GeoGebra model, and if one small detail is incorrect, it could cause the model not to work at all. This could make students very frustrated and make learning really difficult. Along with this, the proposed solution for this goes very in depth, but there are lots of variables involved, and it may be a bit confusing for students to see what is actually going on in the GeoGebra model, to actually be able to learn from this task effectively.
Math Content: 3
The equation used in the solution is pretty complicated, with lots of variables to keep track of, but it does bring in some concepts from statistics, like normal distribution, which could help students see an interesting way to apply statistics.

Objectives:
- Students will learn more about how to work with a statistical distribution, and how to work with things like median, mean, and standard deviation.
- Students will be able to see how varying variables such as time and initial velocity affects density waves

Extensions: The model uses a circular closed loop for a road, and even though that is more convenient for the math and the model, it doesn't seem very realistic. So, if there is a way to model a long stretch of road or an intersection instead of a closed circle, that might be more meaningful for the students' learning.

Karlie Goretski
Rachel Hoard

Task 8.11: Wildfire

Keywords
Code, command buttons, time, direction, probability, variables, meta-scripts

Overall: 3
This task is very difficult for students. It involves an extensive knowledge of using GeoGebra to create scripts and codes to show visual representations of the spread of wildfires from a given direction.
Engaging: 3
Difficulty: 5
Content: 3

Objectives:
- Use Geogebra to create a model that demonstrates how a wildfire may spread based on dryness, wind speed, and other parameters
- Learning to create command buttons to measure time
- Using wind directions to predict the probability to each cell to catch fire from that direction
- Learning to create a script and code on Geogebra that updates the area according to rules created for every time the time variable changes its value
- Create meta-scripts to resemble previous row

Discussion: This task walks students through creating a model to represent the probability of a wildfire spreading to adjacent areas based on parameters and wind directions. This allows students to become familiar using scripts and coding to start and stop an animation in GeoGebra. However, this task is likely to be very difficult for students especially if they are not comfortable using GeoGebra. The task can also be done using a script in JavaScript.

Anticipated Student Difficulties:
- Creating a spreadsheet in GeoGebra
- Only create one spreadsheet instead of three
- Creating a Graphical User Interface to control time and reset the animation
- Representing the barriers including roads and rivers that the fire cannot pass by using a 0 in each cell in the spreadsheet
- Constructing a square to represent the spreadsheet as an untouched fire
- Constructing time control buttons to control the animation
- Creating scripts to code to create a representation of the spread of wildfires

Technology:
GeoGebra

Index